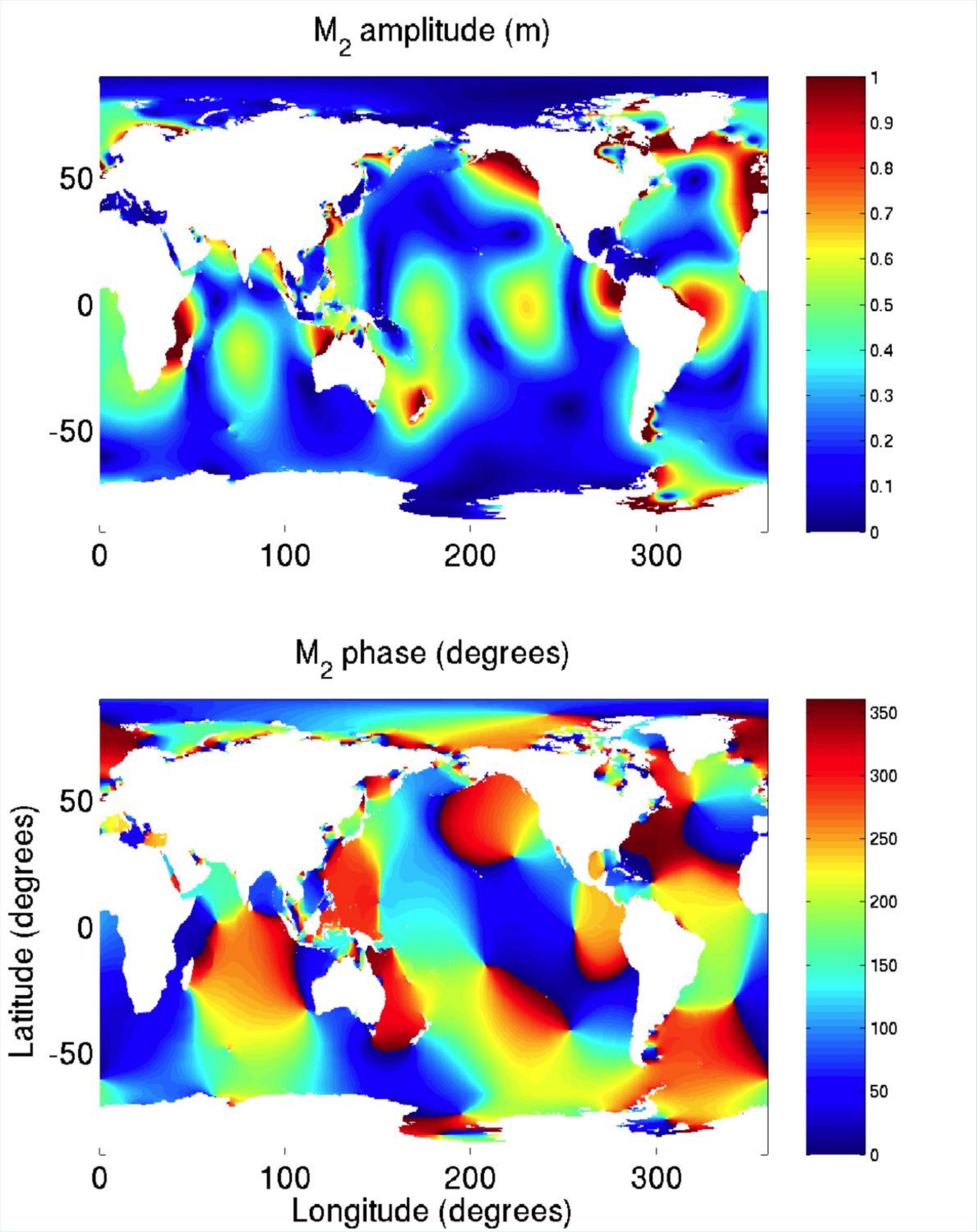
Tides Supplement

Brian Arbic



OVERVIEW QUESTIONS

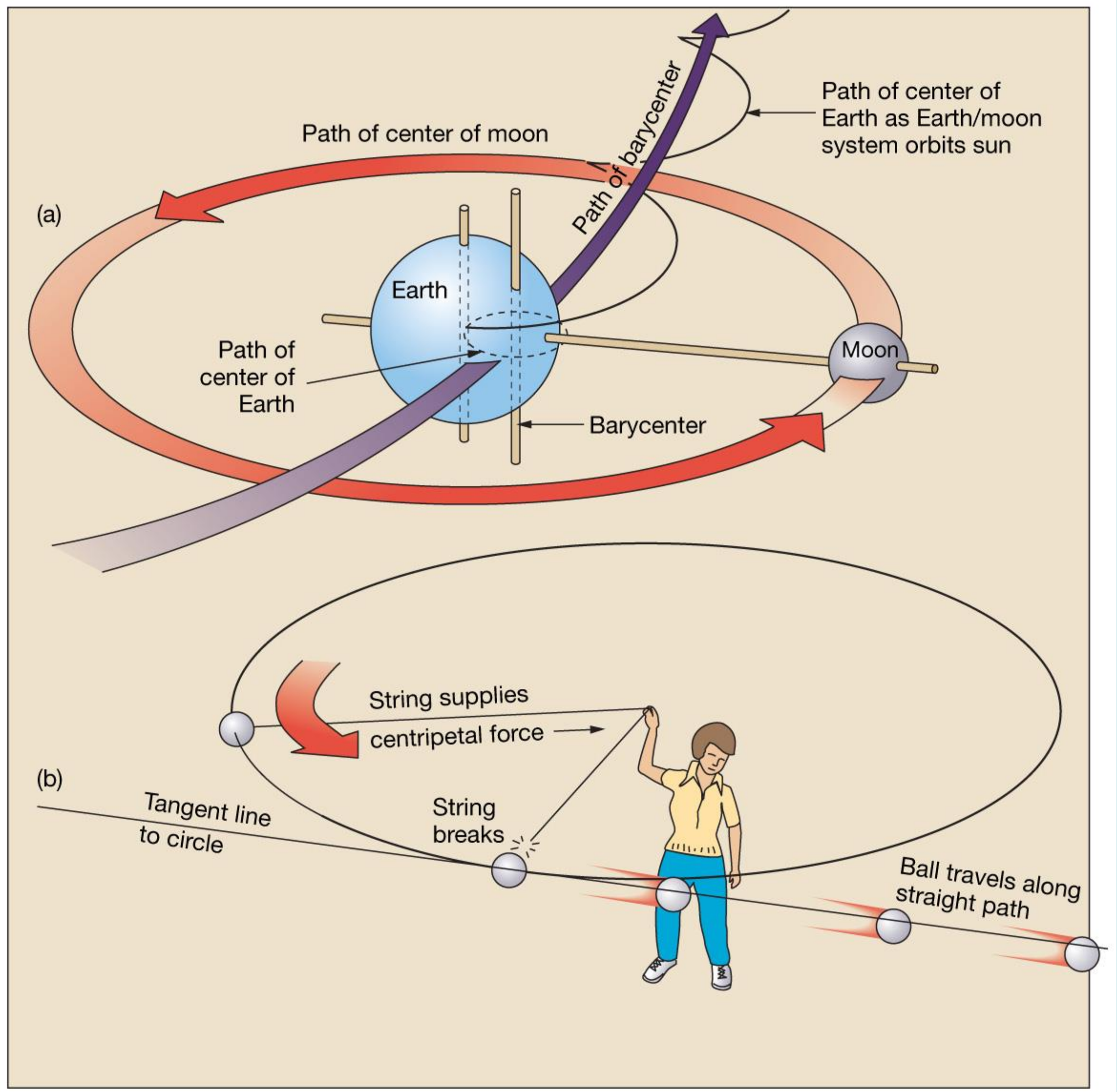
- What causes the tides?
- How is a lunar day different from a solar day?
- Which body creates a larger tidal influence on Earth: the Moon or the Sun?
- How do the relative positions of the Earth-Moon-Sun affect the tidal range on Earth?
- What are differences between diurnal, semidiurnal, and mixed tidal patterns?
- What do tides really look like in the ocean?
- What tools are used in modern tidal research?

What causes the tides?

- Tides are very long and regular shallow-water waves, causing periodic (predictable) changes in sea level and currents.
- The lunar equilibrium tide (idealized tide) is due to the *difference* between the gravitational attraction of the Moon on the ocean and the centripetal forces due to orbital rotation of the Moon and Earth around their common barycenter (center of mass)
- The same physics also apply to the Earth-Sun system, thus yielding a solar equilibrium tide
- The actual tides are a complex response of the ocean system—involving friction, rotation, the shape of the seafloor, the placement of continents, and motions of the solid earth—to the much simpler equilibrium tide.

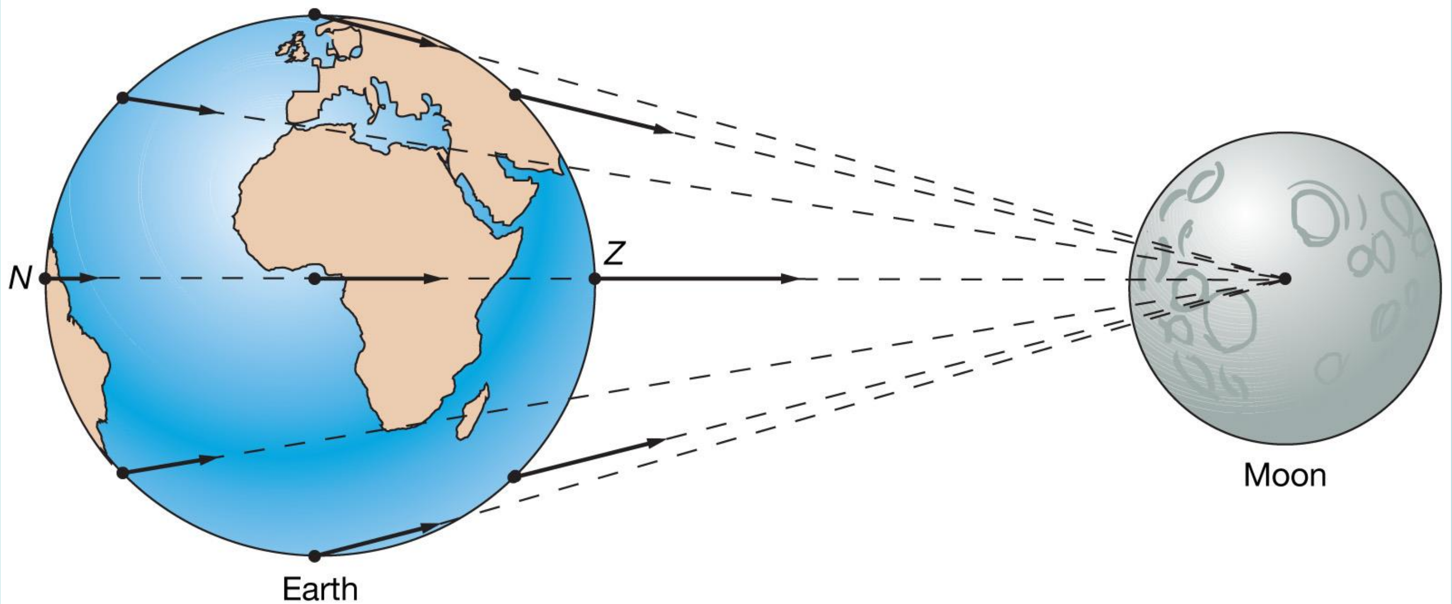
Earth-Moon system rotation

- Barycenter between Moon and Earth revolves around Sun



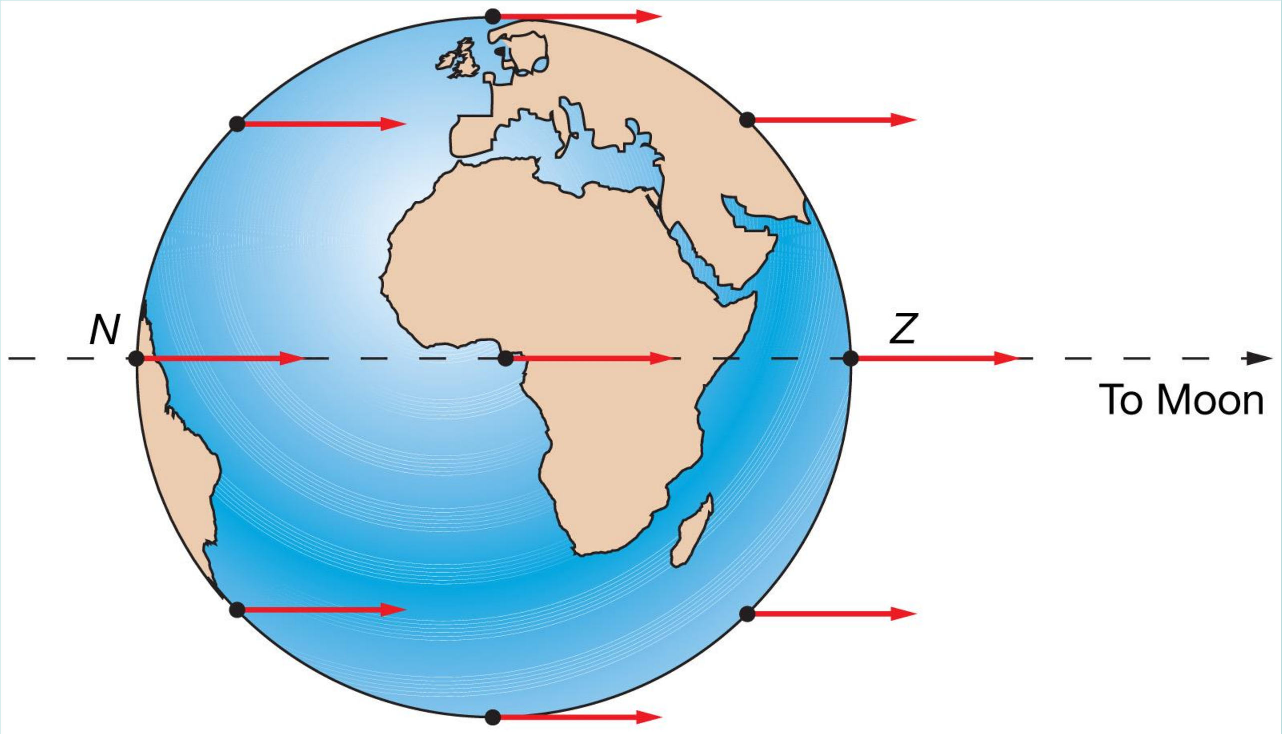
Gravitational forces

- Every particle attracts every other particle
- Gravitational force proportional to product of masses
- Inversely proportional to square of separation distance r
- $F=GM_1M_2/r^2$
- The gravitational force is largest on the side of the Earth closer to the moon and least on the side further from the Moon. The force is always directed toward the center of the Moon.

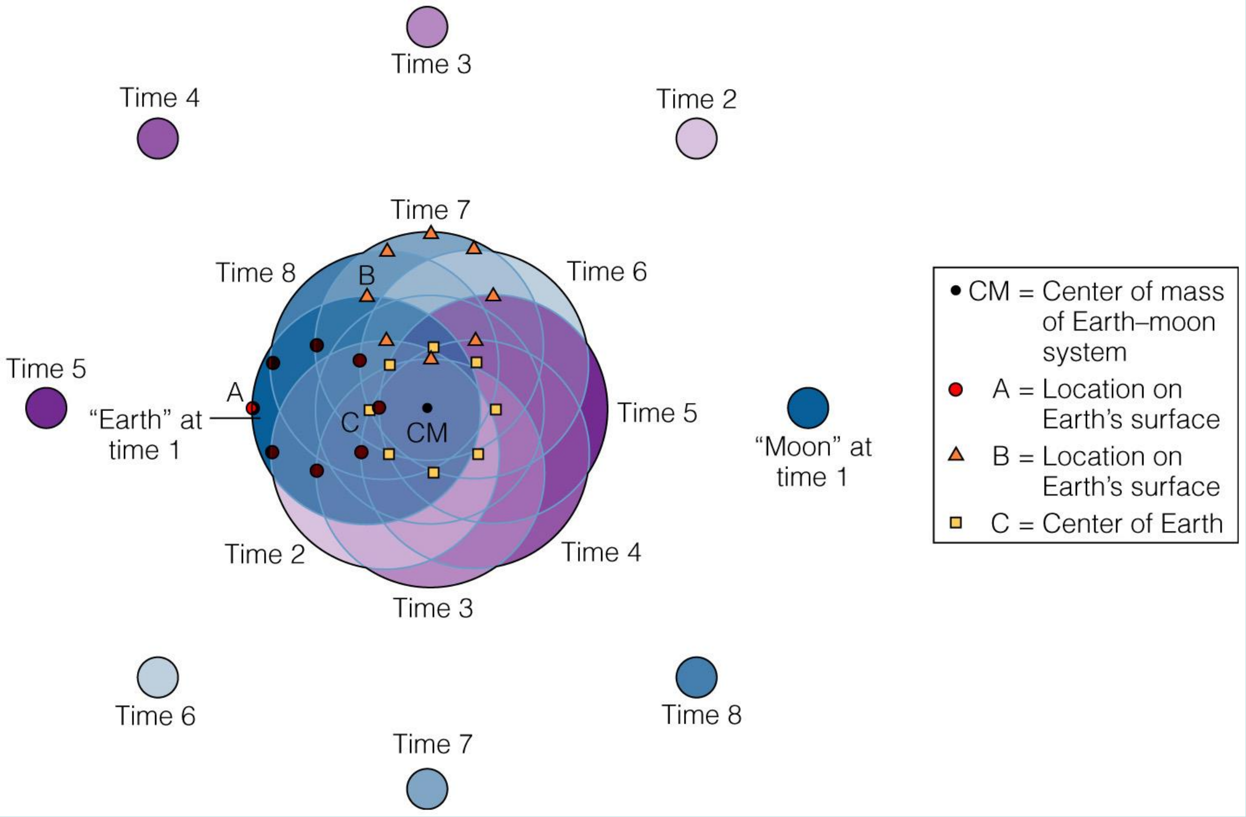


Centripetal force

- Center-seeking force; rotation about “barycenter” or center of mass between two bodies in orbital motion.
- Tethers Earth and Moon to each other
- The centripetal force (the red arrows) is everywhere the same. The red arrows are all the same length and point in in the same direction.



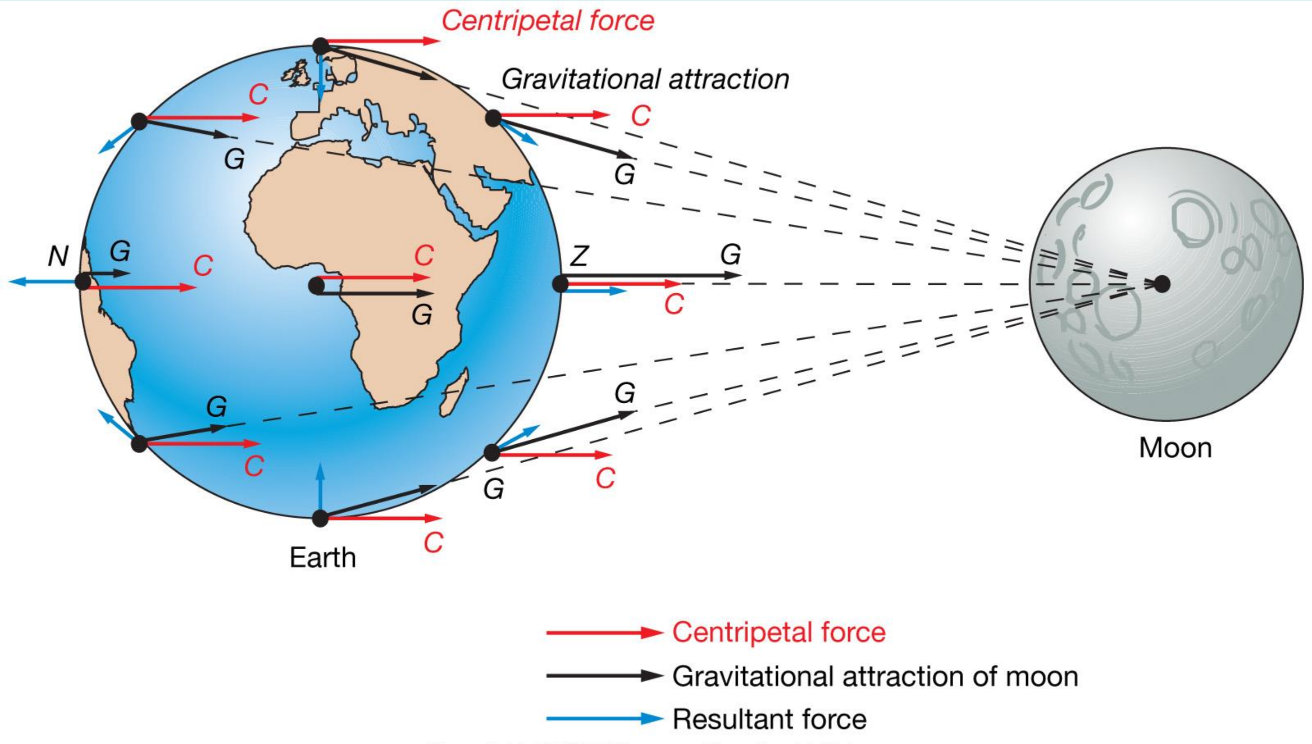
Distribution of centripetal force



Each particle on earth moves around the center of mass (CM) in circles of same diameter, in same amount of time → each particle experiences same centripetal force

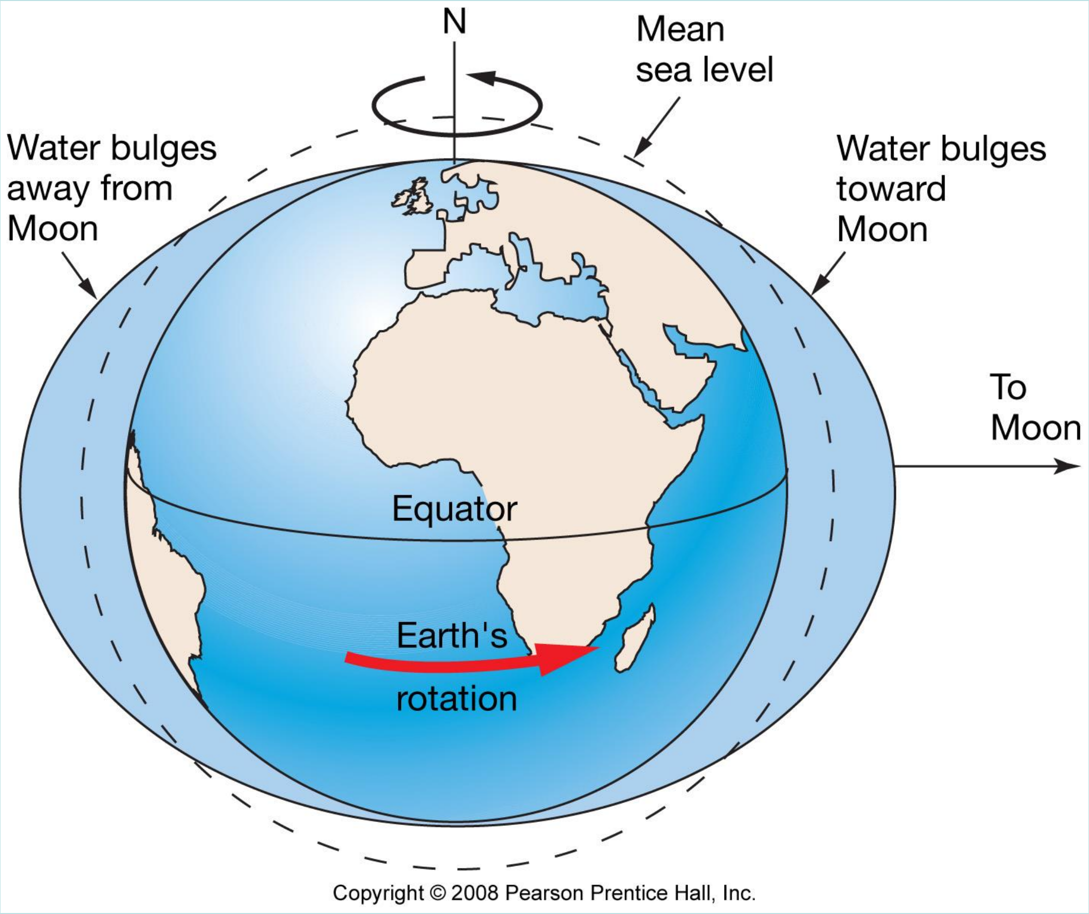
Tide-producing (resultant) forces

- Resultant forces = differences between centripetal and gravitational forces; proportional to $1/r^3$
- The **tidal generating force** (blue) is the **difference** of the **Moon's gravitational force** (black) and the **centripetal force** (red). Tide-generating forces point towards the Moon at Z (zenith) and away from the Moon at N (nadir).



Tidal bulges (lunar): Ideal Earth covered by ocean

- One bulge faces Moon
- Other bulge on opposite side
- Tidal cycle results from earth rotating underneath these bulges→ two high tides and two low tides during a day
- The simple shape in the figure is called the equilibrium tide, or idealized tide.
- Actual tide in oceans is a complex response to forcing of equilibrium tide



Which body creates a larger tidal influence on Earth: the Moon or the Sun?

- The same reasoning we have just applied to the Moon can also be applied to the Sun.
- The Sun is much more massive than the Moon, but also much further away. The effect on tides goes as mass over the distance-cubed (M/r^3).
- Mass of Sun = 27,000,000 X Mass of Moon
- Distance Earth→Sun = 390 X Distance Earth→Moon
- In-class exercise: How large is the solar equilibrium tide compared to the lunar equilibrium tide?

Which body creates a larger tidal influence on Earth: the Moon or the Sun?

- Mass of Sun = 27,000,000 X Mass of Moon
- Distance Earth→Sun = 390 X Distance Earth→Moon
- 27,000,000 / (390^3) = 0.45
- The Sun is much more massive than the Moon, but also much farther away
- The tidal force of the Sun on Earth is therefore about ½ that of the Moon. So the tidal bulges due to the Sun are about half that of the Moon.

Background to in-class exercise: definition of a day

- Observers at a fixed point on Earth will see two high tides and two low tides during a “day”.
- A “day” is the time it takes for the Earth to rotate once about its axis.
- The rotation is measured with respect to astronomical objects.
- Solar day: time for an Earth-bound observer to turn and see the same point on the Sun again.
- Sidereal day: measured against the distant stars.
- Lunar day: measured against the Moon.

In-class exercise: definition of a day

- On google, type “solar day wolfram research” and then “sidereal day wolfram research”.
- Read the articles which are pulled up:

http://scienceworld.**wolfram.com/astronomy/SolarDay.html**

and

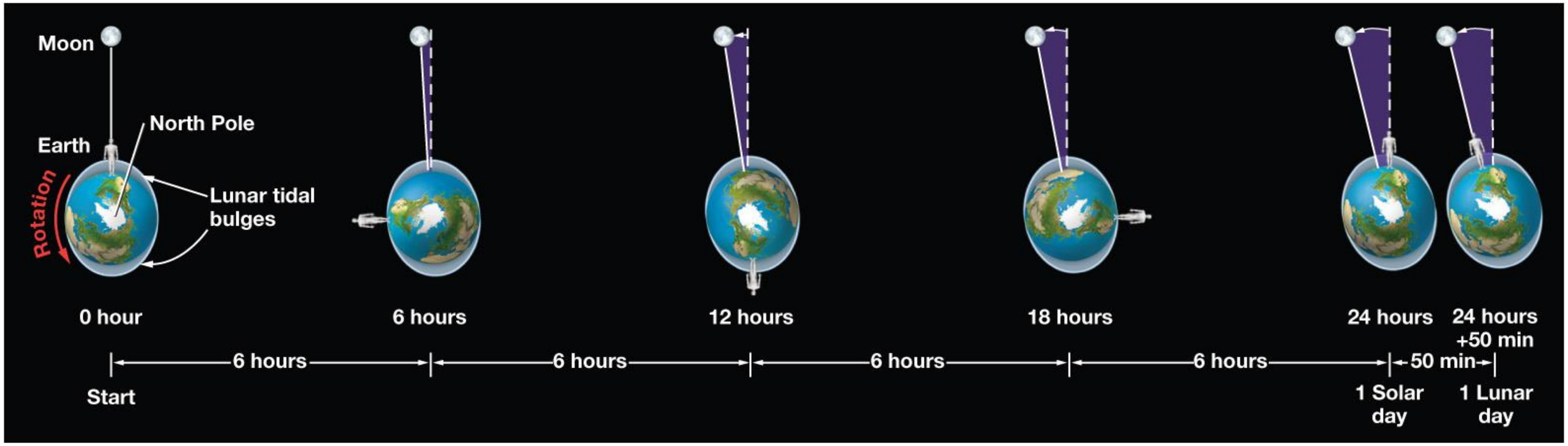
http://scienceworld.**wolfram.com/astronomy/SiderealDay.html**

- On google, type “animation of lunar day noaa”.
- Read the article which is pulled up:

http://oceanservice.**noaa.gov/education/kits/tides/media/supp_tide05.html**

How is a lunar day different from a solar day?

- Moon orbits Earth
- While the Earth rotates around its axis, the Moon moves relative to the Earth, so
- 24 hours 50 minutes for observer to see subsequent Moons directly overhead (lunar day)
- High lunar tides are 12 hours and 25 minutes apart → period of principal lunar semidiurnal tide (M_2) is 12 h 25 m
- 24 hours for observer to see subsequent Suns directly overhead (solar day)
- High solar tides are 12 hours apart → period of principal solar semidiurnal tide (S_2) is 12 h
- How long would it take to see subsequent distant stars directly overhead? (sidereal day—23 h 56 m)



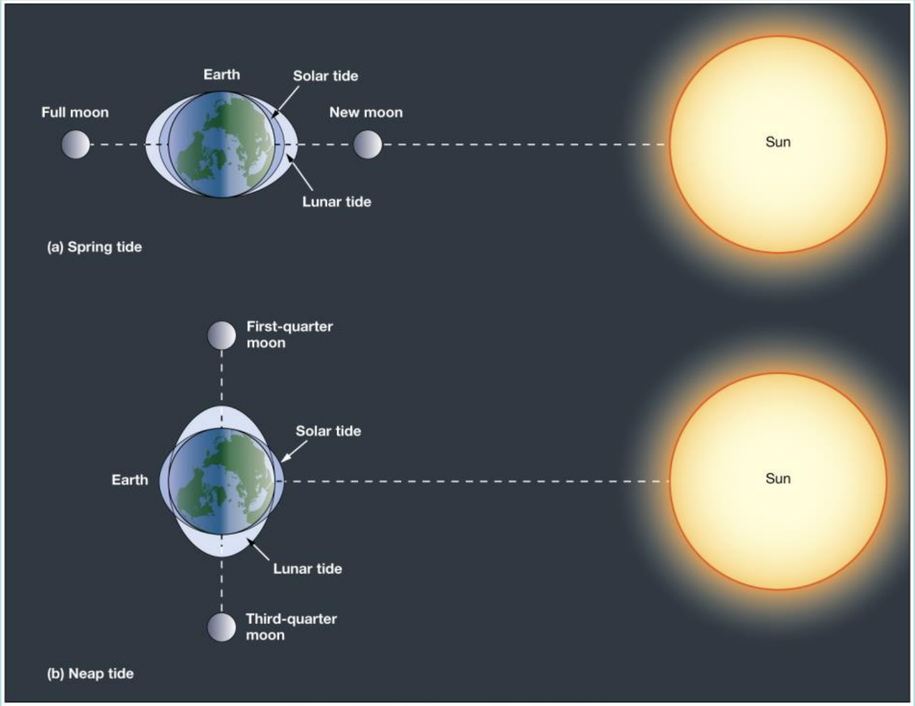
How do the relative positions of Earth-Moon-Sun affect the tidal ranges on Earth?

- Answer shown on next slide

Spring tides occur when the Moon, Earth and Sun are aligned (in syzygy, upper). Either new moon or full moon. Tidal range (difference between high and low tides) is large: exceptionally high high tides, low low tides.

Neap tides occur when Earth-Moon line is at right angles to Earth-Sun line (lower). Sun's bulges do not reinforce the Moon's bulges then. Tidal range is lower; high tides not so high, and low tides not so low. Occurs during first-quarter or third-quarter moon.

Spring tides occur twice a (lunar) month (not during Spring Season!!). Similarly, the neap tides occur twice a (lunar) month; twice each 29.5 days.

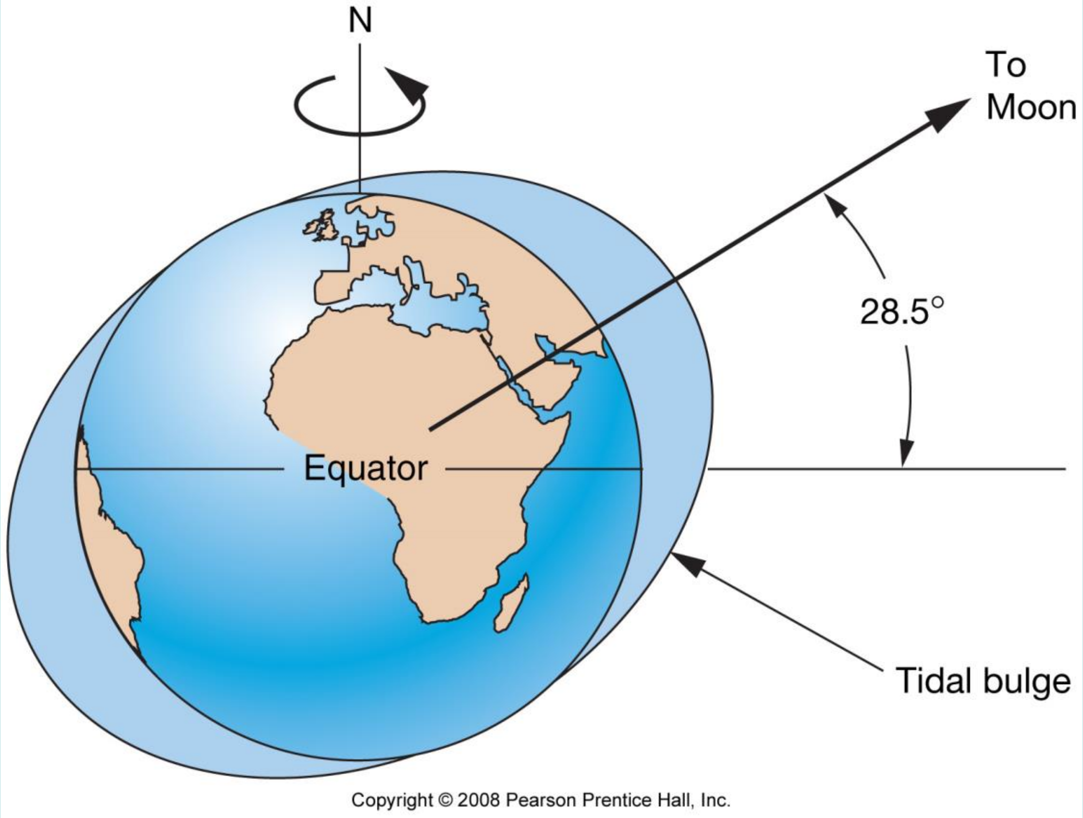


What are differences between diurnal, semidiurnal, and mixed tidal patterns?

- Semidiurnal tides have (approximately) two cycles per day (two highs and two lows)—in other words, a *period* near 12 hours
- Diurnal tides have approximately one cycle per day—i.e., a *period* near 24 hours. Diurnal tides are caused by the *declination* of the Moon's orbit.
- Semidiurnal tides are common along the US Atlantic Coast.
- Some locations (including in the Gulf of Mexico) experience predominantly diurnal tides
- In most locations the tides are mixed semidiurnal-diurnal.

Declination

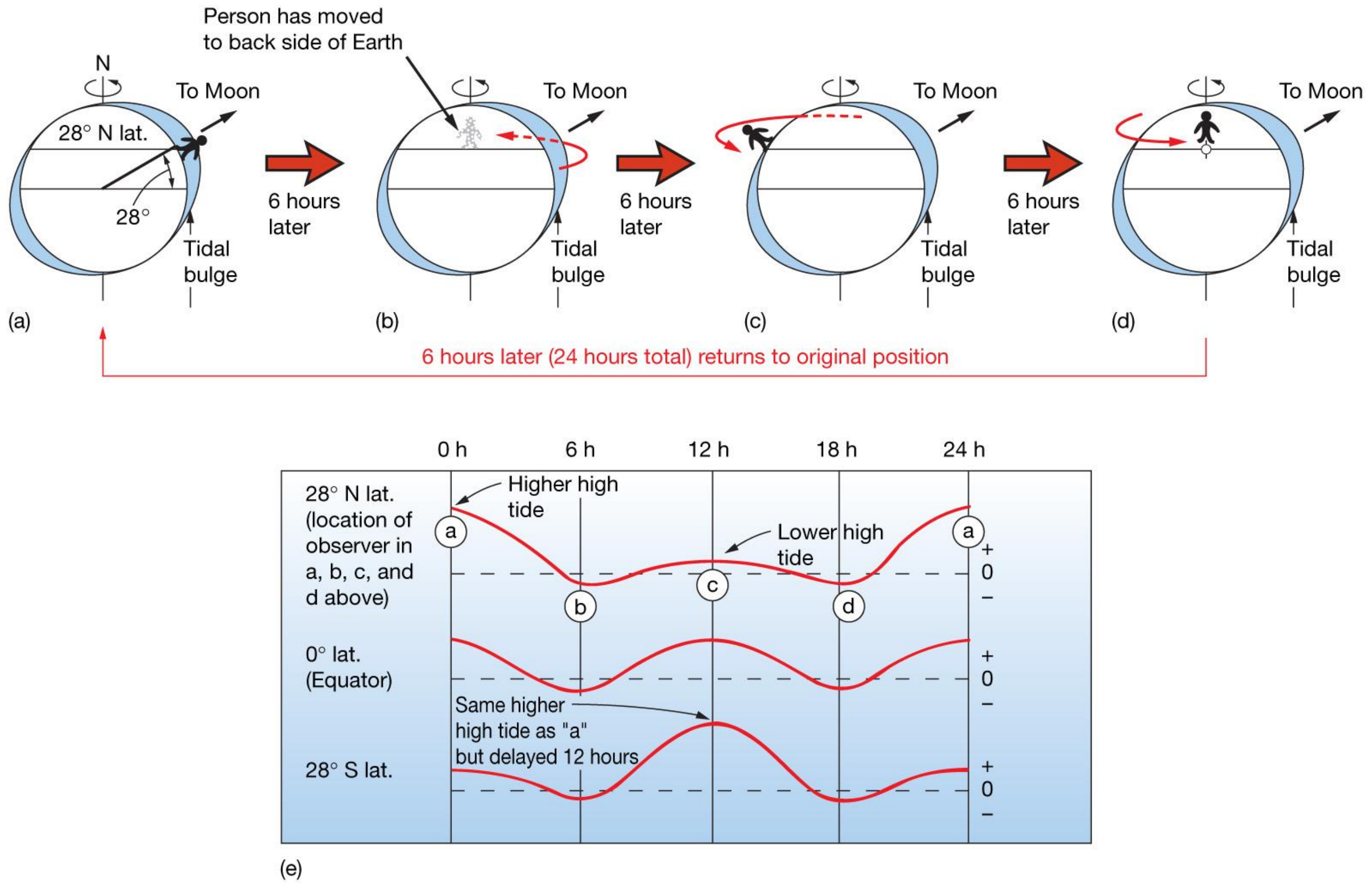
- Angular distance of Moon or Sun above or below Earth's equator
- Sun to Earth: 23.5° N or S of equator
- Moon to Earth: 28.5° N or S of equator



- Shifts lunar and solar bulges from equator
- Diurnal inequality: Two high tides a day are of unequal size → Tidal forcing exists at periods near 24 h

Declination and tides

- Predicted idealized tides (unequal, but symmetric tidal ranges)



Summary of equilibrium tidal theory

- Lunar equilibrium tide has two bulges, one on side of Earth facing Moon, one on opposite side of Earth
- Solar equilibrium tide has two bulges, one facing Sun, one on opposite side of Earth
- Lunar bulge about twice as large as solar bulge
- Spring tides occur when solar and lunar bulges are lined up, neap tides occur when the bulges work against each other
- Lunar tides delayed by 50 minutes every day. If for example the high tide today is at 15:00 (3 PM), then tomorrow (two tidal cycles later) it will be at 15:50 (3:50 pm)
- Declination leads to diurnal tides

In-class exercise: can the actual ocean tides be in equilibrium?

- Recall from the tsunami exercise that the phase speed of shallow water waves in a 4000 m ocean is about 200 m/s
- What is the speed that waves would need to stay under the moon as it orbits around the Earth?
- Compute $2\pi R$/(24 hours 50 minutes), where R is the Earth's radius. Put your answer in units of m/s. Is the velocity you obtain faster than or slower than the speed of shallow water waves?

What do tides really look like in the ocean?

- Tides in the actual ocean are much more complicated than the simple bulges (equilibrium tide, also called idealized tide) we have described thus far
- The actual tide is a *dynamical* response of the oceans to the equilibrium forcing, and does not look like the equilibrium tide
  - Tides move as shallow-water waves, but the ocean depth is not deep enough for the waves to keep up with the positions of the Moon and Sun.
  - Continents get in the way.
  - Coriolis effect is important.
  - Shape of seafloor is important, since wave speed depends on depth.
  - Frictional effects control the strength of the tides.
  - Solid earth also has a tide; this affects ocean tides because it changes the seafloor which tides are measured against, as well as the gravity field.

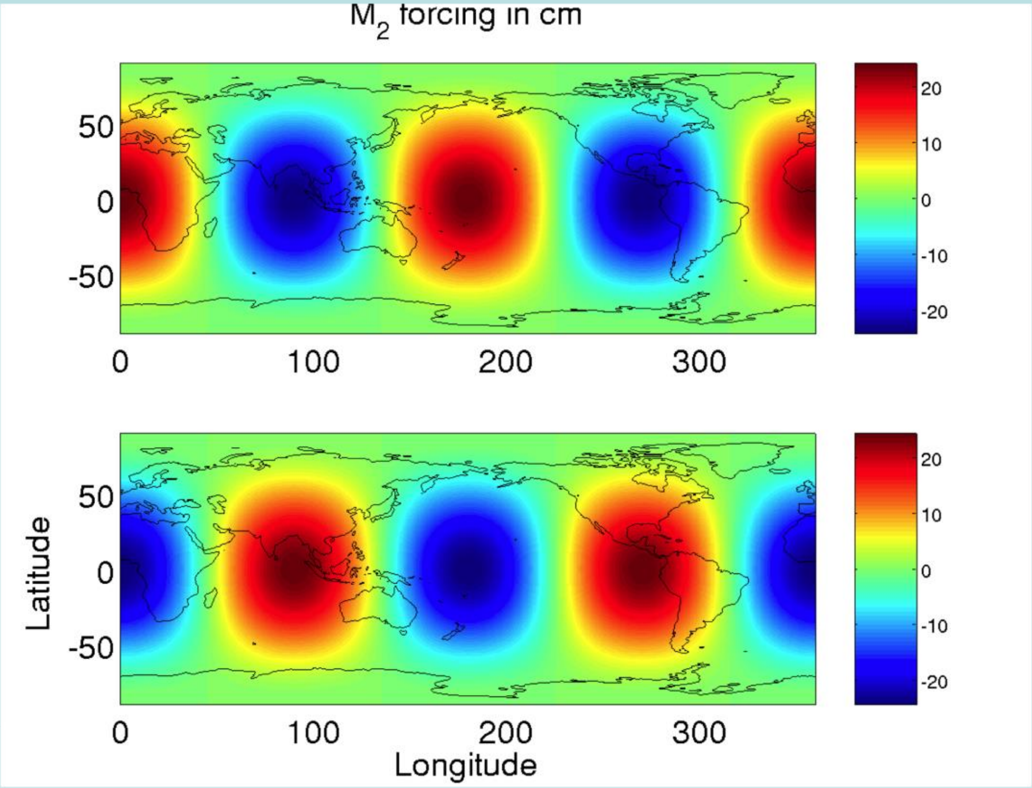
What tools are used in modern tidal research?

- Research into tides was revolutionized by the launch of satellite altimeters, which can accurately measure tidal sea surface elevations from orbit.
- The Apollo program of the 1960's provided another advance, with mirrors left on the Moon by astronauts.
  - Distance from Earth to Moon is increasing by 4 cm/year, from which the tidal energy dissipation can be accurately inferred.
- Another important advance in tidal research (and ocean research in general) has been the ever-increasing power of modern computers, which enable models of the ocean tides to be done at higher and higher resolution.

Animation of tidal sea surface heights

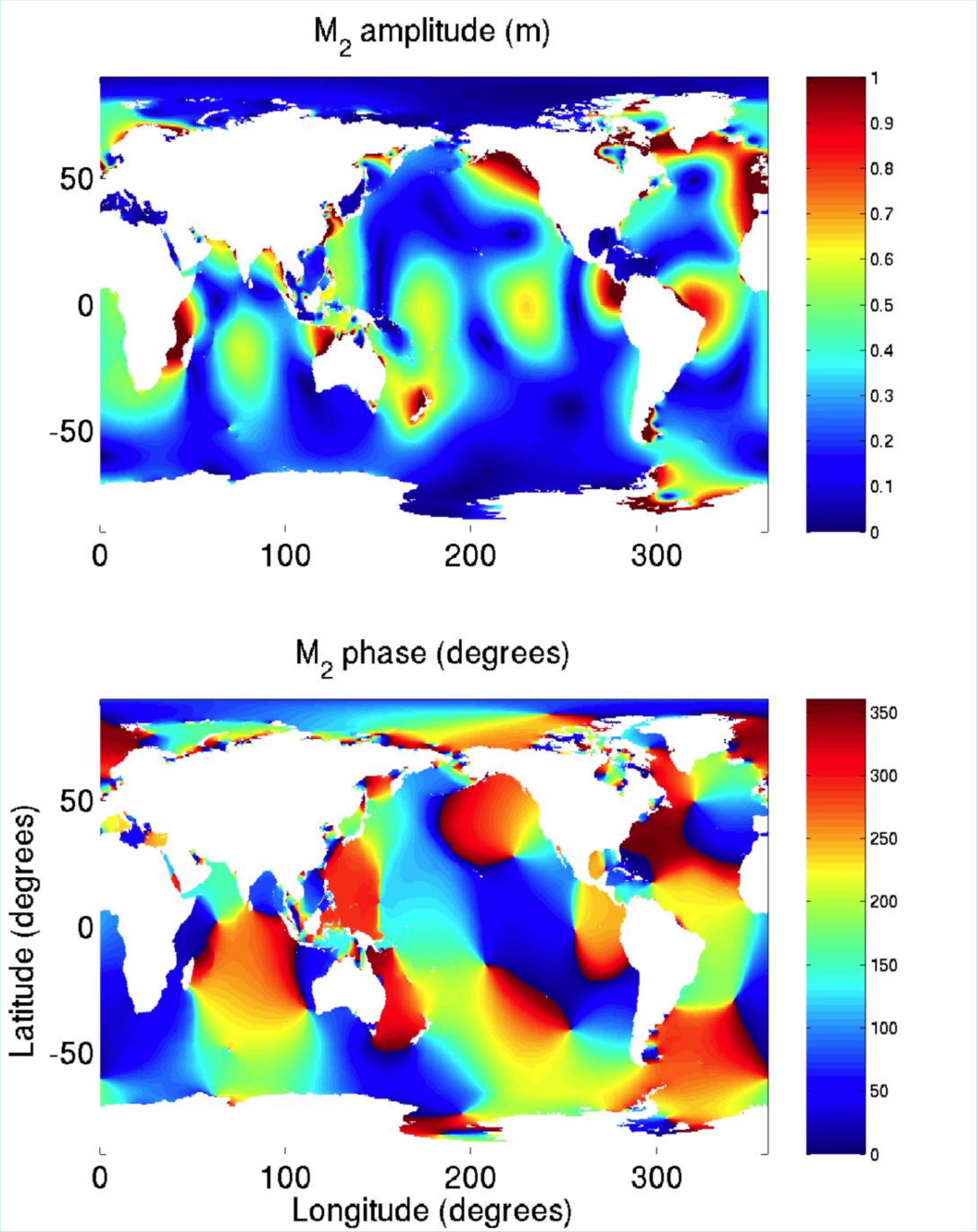
- We will now look at one of the most widely used numerical (computer) models of tidal sea surface heights.
- The model assimilates satellite altimetry data.
- Typing “TPXO” in google leads to http://volkov.oce.orst.edu/tides

This plot shows the equilibrium tidal forcing of the principal lunar semidiurnal tide (M_2), 6 hours 12 minutes apart (half a cycle)



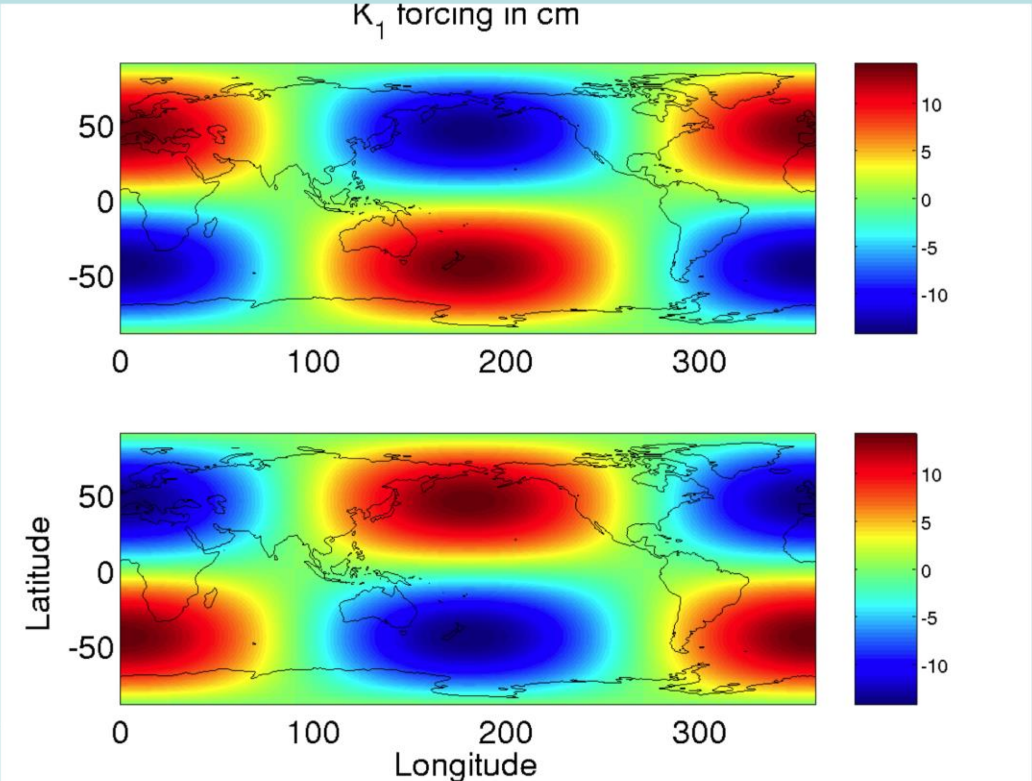
This plot shows the amplitude and phase of the principal lunar semidiurnal tide (M_2) in the ocean, as measured by satellite altimeters

Note that the principal solar semidiurnal tide (S_2) has a similar pattern but is about half as large, and has a period of 12 hours not 12 hours 25 minutes

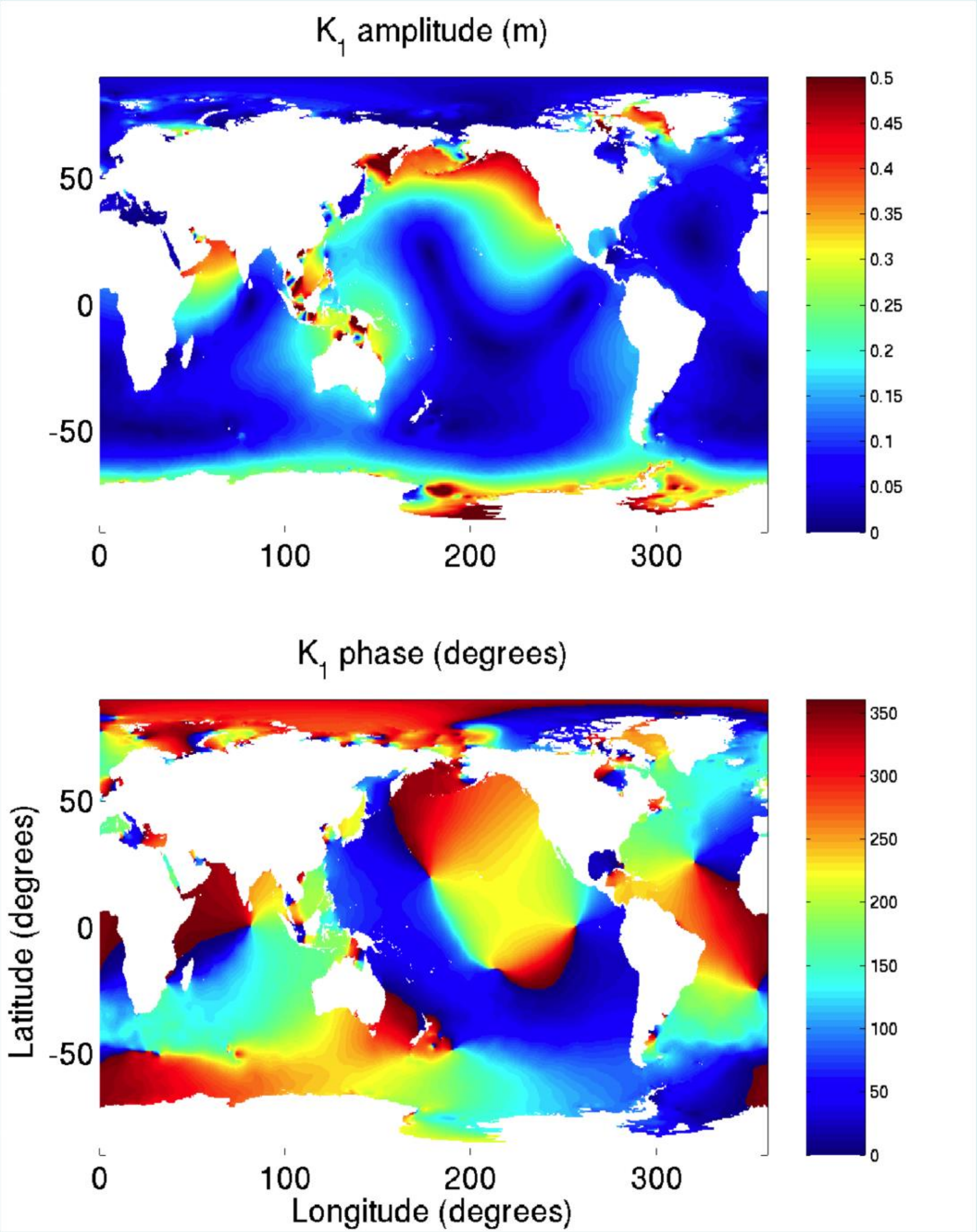


This plot shows the equilibrium tidal forcing of the largest diurnal tide in the world ocean (K_1), 11 hours 58 minutes apart (half a cycle)

Note that the period of K_1 is 23 hours 56 minutes = 1 sidereal day (time for observer to see subsequent distant star directly overhead)

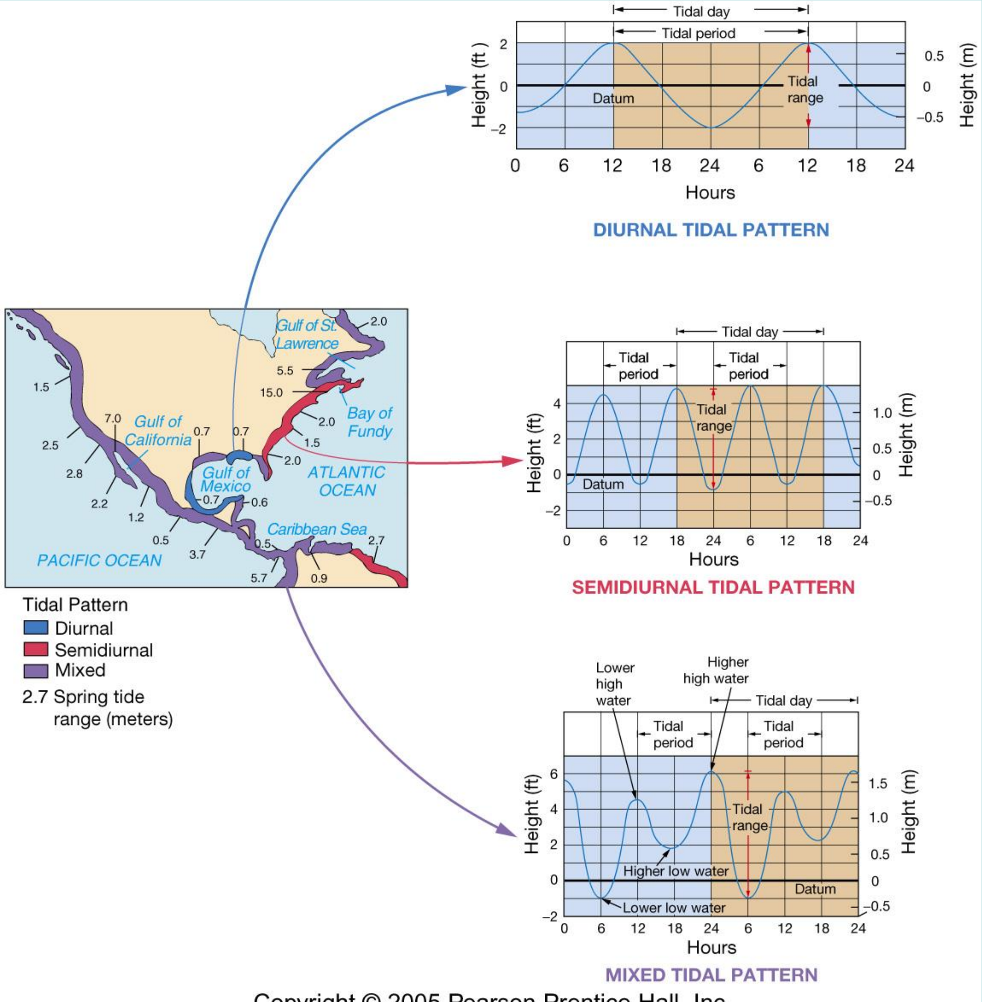


This plot shows the amplitude and phase of the largest diurnal tide in the ocean (K_1), as measured by satellite altimeters

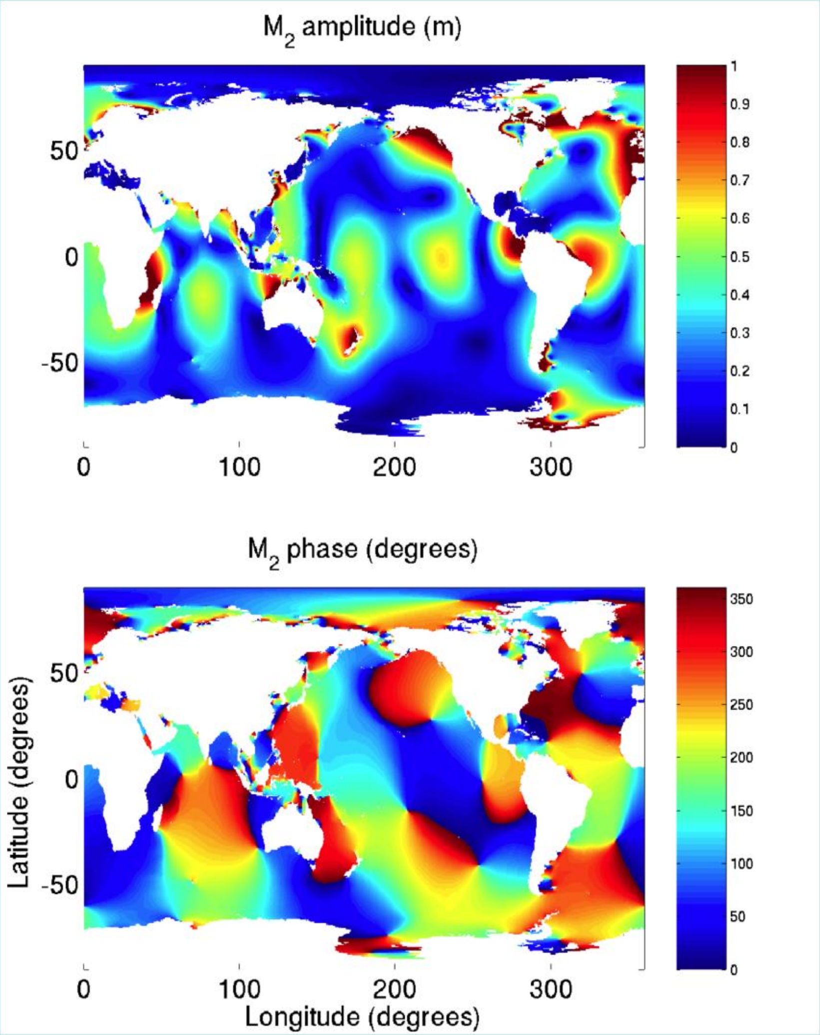


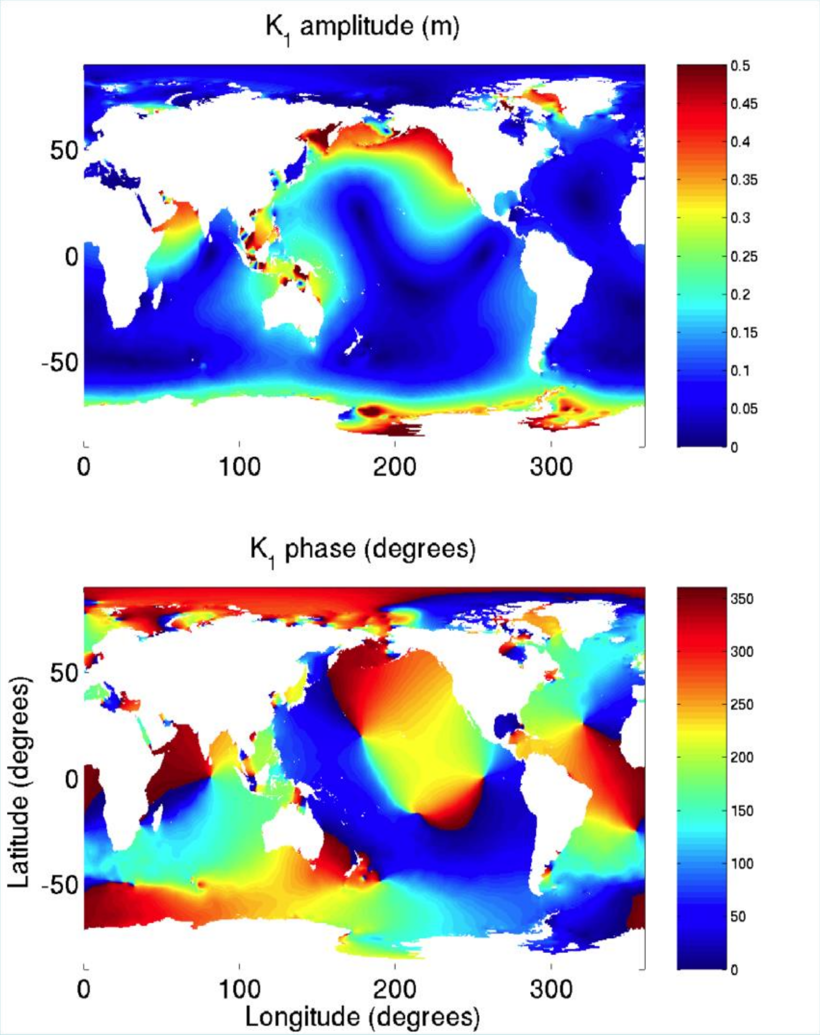
Tidal Patterns

- In Gulf of Mexico we have diurnal tides in some places (e.g. Pensacola) and mixed tides elsewhere
- On US east coast: semi-diurnal tides
- On US west coast: mixed tides



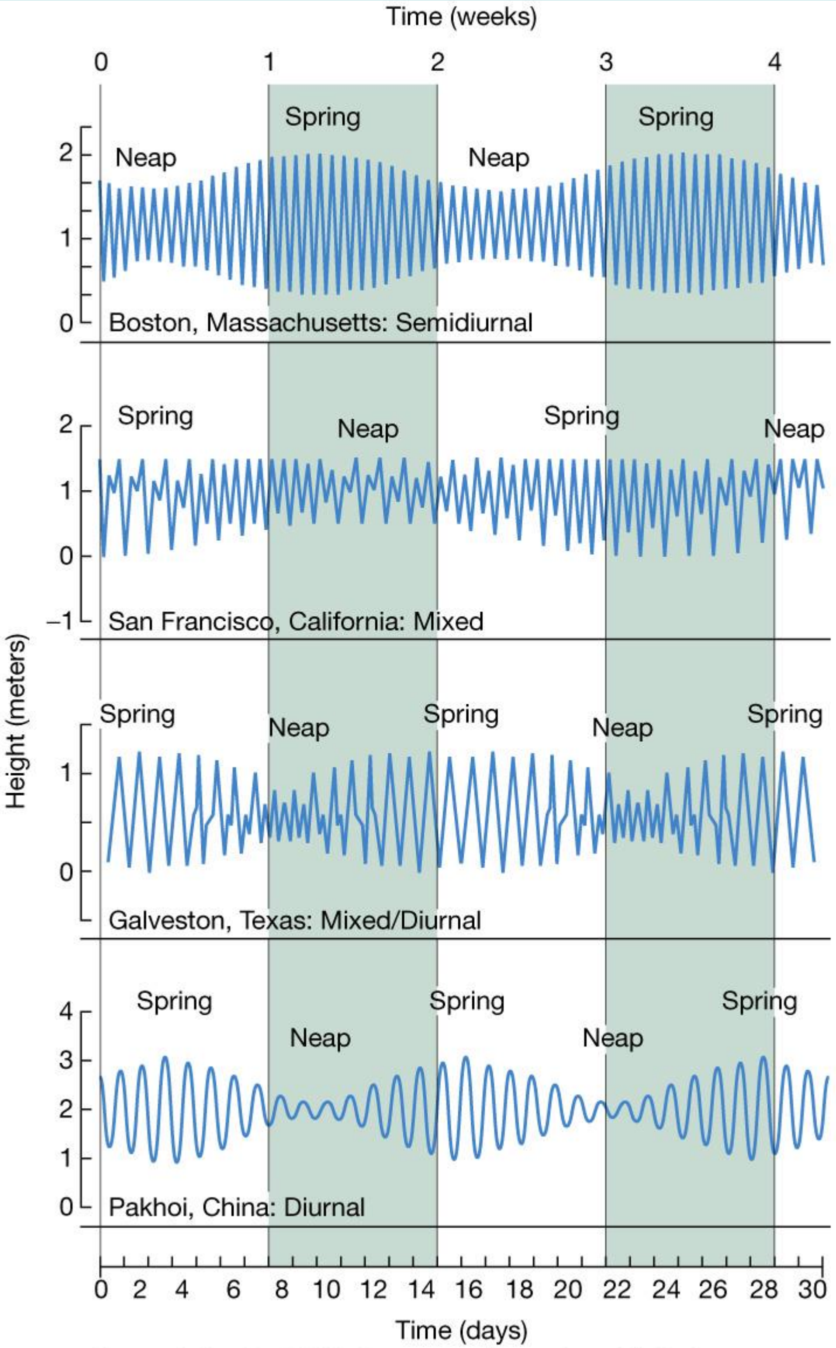
The tidal patterns (mixed vs. semidiurnal vs. diurnal) can be understood by comparing the upper parts of these two plots:





Monthly tidal curves

- During a month there are two spring tides and two neap tides at all locations.
- Boston has semi-diurnal tides
- San Francisco and Galveston have diurnal/mixed tides
- Pakhoi, China has diurnal tides



Summary—Tides I

- The lunar equilibrium tide (idealized tide) is due to the *difference* between the gravitational attraction of the Moon on the ocean and the centripetal forces due to orbital rotation of Moon and Earth around their common barycenter (center of mass)
- The same physics applied to the Sun's gravitational attraction yields a solar equilibrium tide
- When the Sun, Earth, and Moon are all in a line, the tidal forces of the Sun and Moon reinforce, and the ocean experiences spring tides (very high high tides, very low low tides). When the Moon is in the first- or third-quarter, the tidal bulges produced by the Sun are at right angles to those generated by the Moon, and the ocean experiences neap tides (smaller range between high and low tides).
- The actual tides are a complex response of the ocean system—involving friction, rotation, the shape of the seafloor, the placement of continents, and motions of the solid earth—to the much simpler equilibrium tide.
- Most locations experience a mixed tidal pattern, in which both semidiurnal and diurnal tides are strong. In some locations, for instance parts of the Gulf of Mexico, diurnal tides dominate, while in others, for instance the US Atlantic coast, semidiurnal tides dominate.
- The advent of satellite altimetry and computer modeling have revolutionized tidal research.