Intro to Physical Oceanography
Lecture 2: Turbulence and mixing

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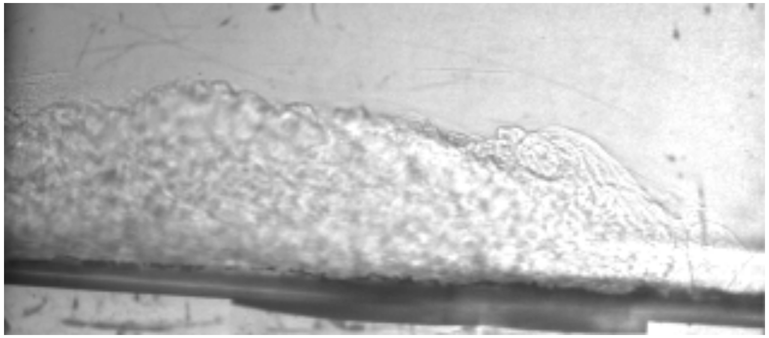
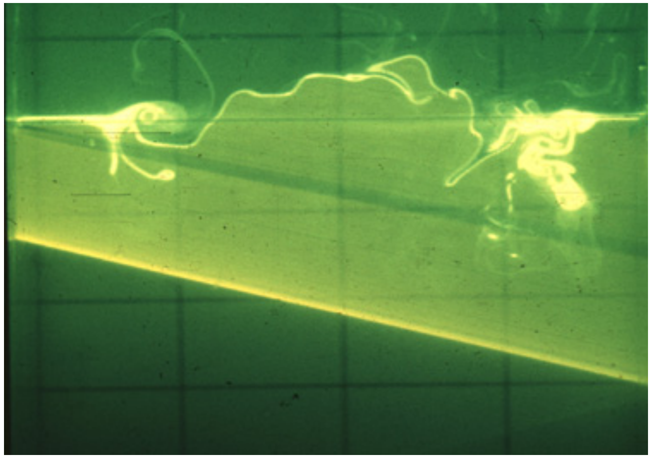
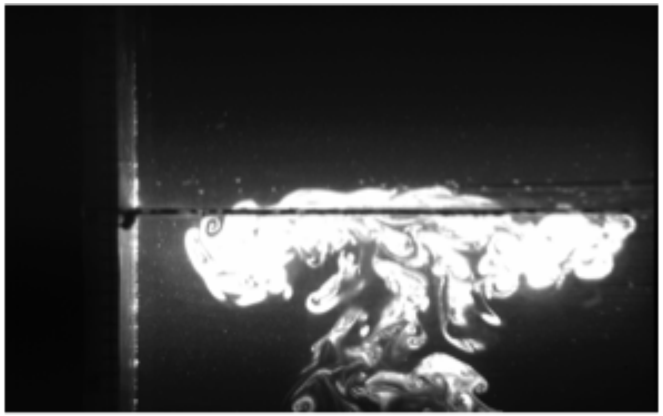


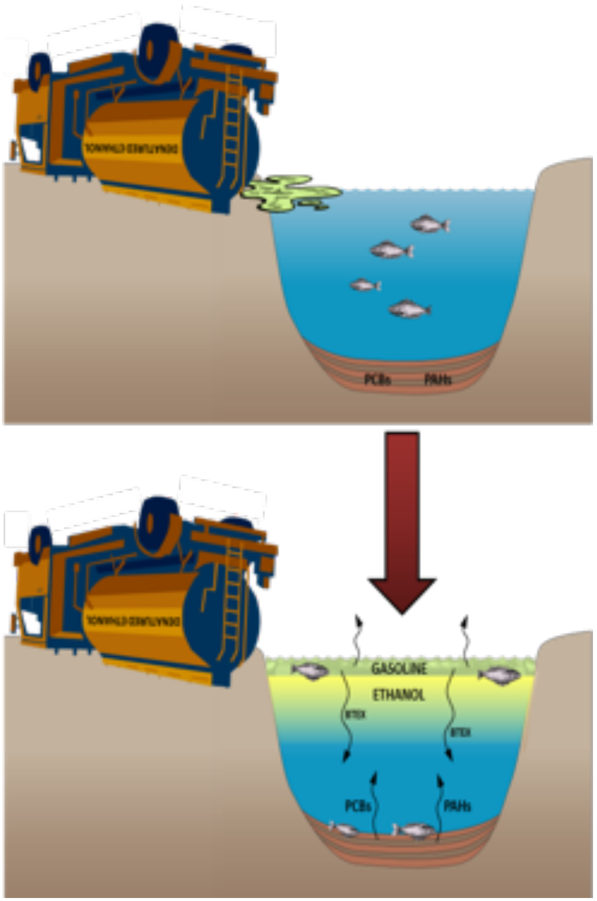
Fluid dynamics

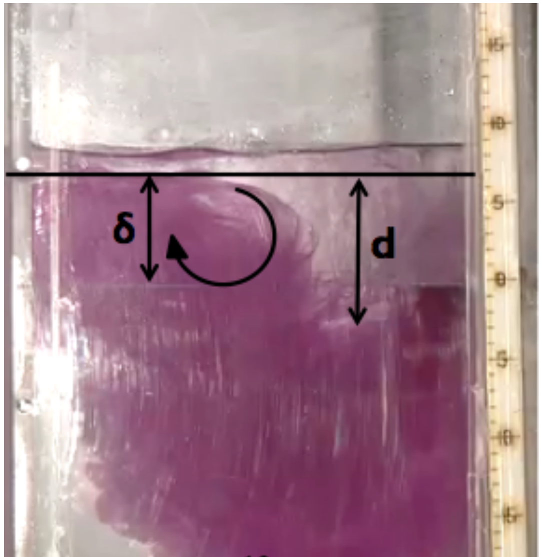


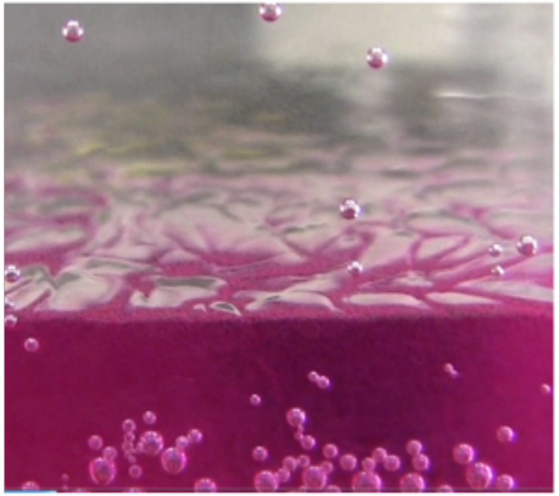


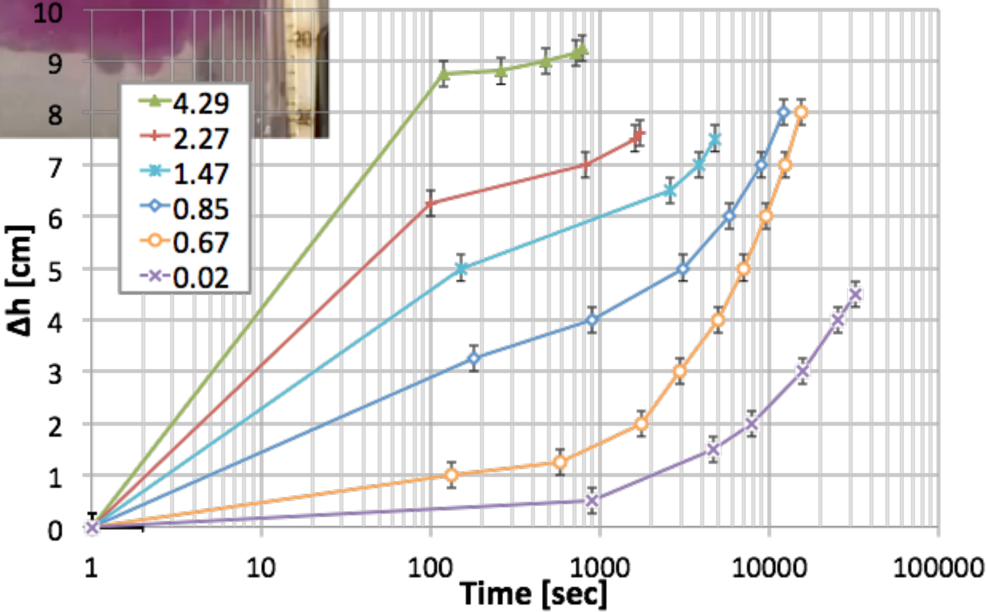
Environmental Fluid Dynamics

Mixing Dynamics Between Water and Biofuels







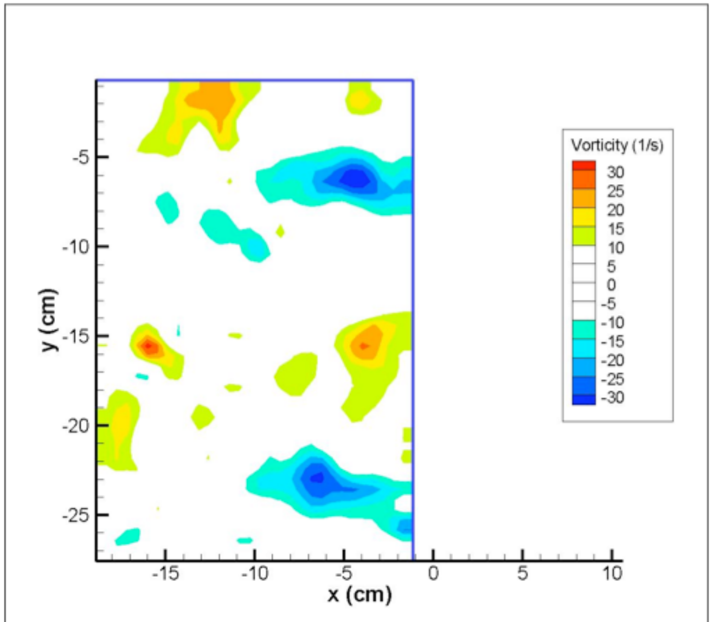


Biological Fluid Dynamics

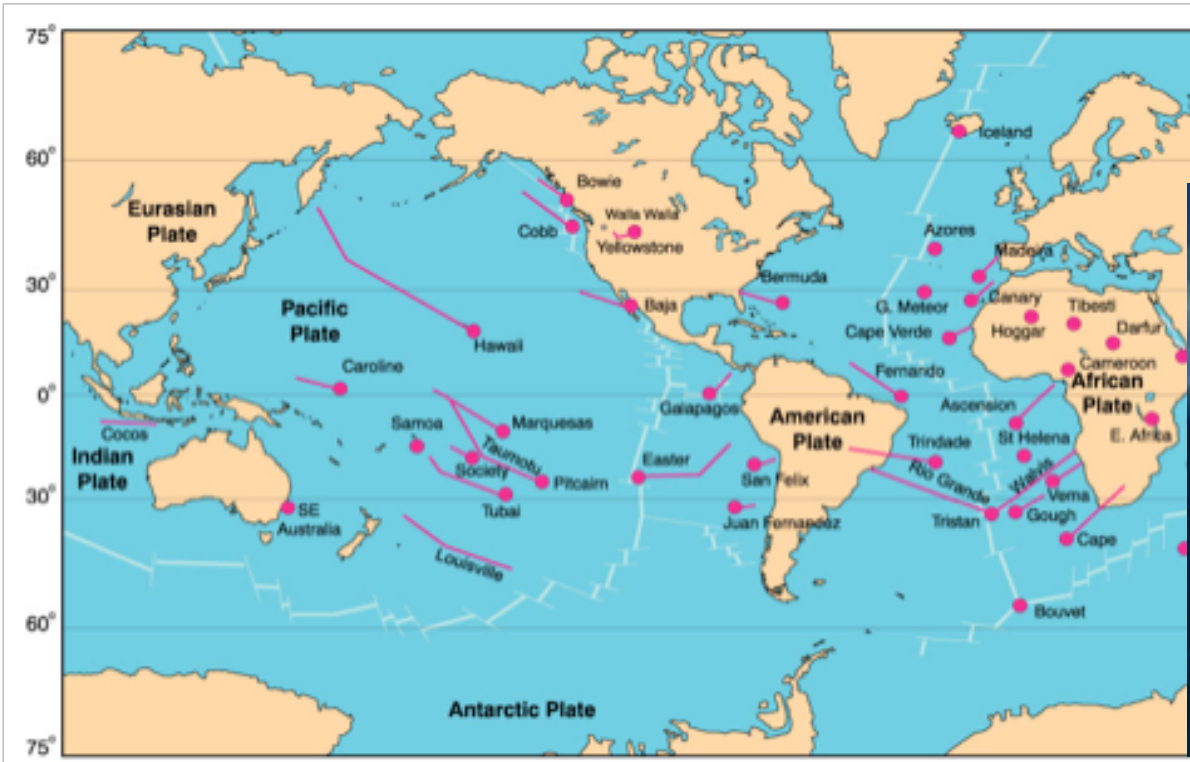
Understanding interactions between fish and turbulence
-> explains reductions in swimming ability, habitat choices, increased performance.

↓

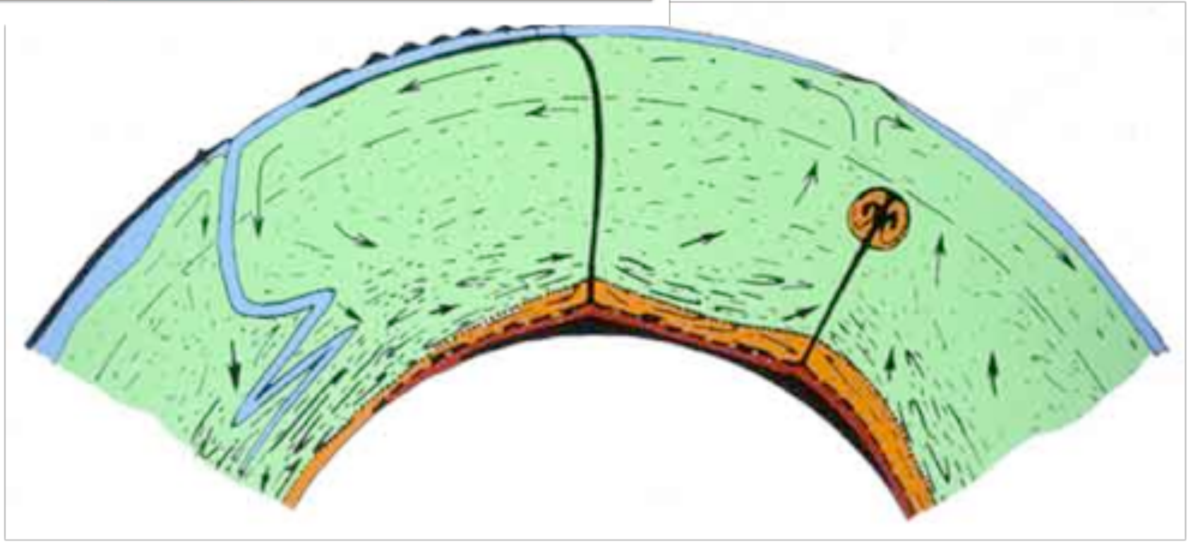
Stream and shoreline restoration, fishway and culvert design, fish swimming

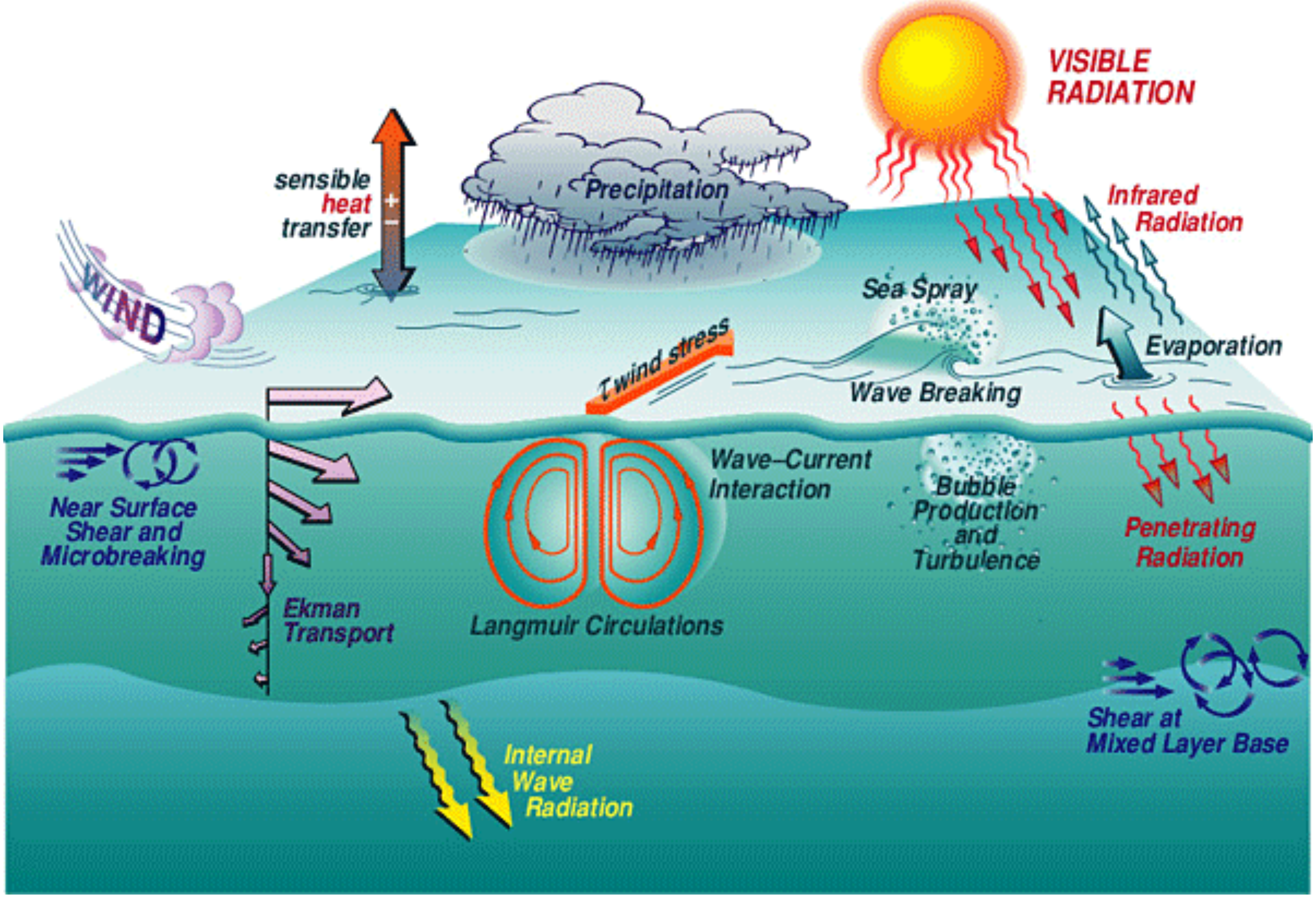


Geophysical Fluid Dynamics



Concentrated volcanic activity.
Chemistry of erupted lavas is significantly different than Mid Ocean Ridges for example





Schematic drawing of processes that contribute to variability in the ocean surface boundary layer.
[R. Weller, WHOI]

Outline

1. Diffusion versus turbulence
2. Laminar and turbulent flows
3. Turbulent flows properties and characteristics

Molecular diffusion

Diffusivity is measured in terms of area per unit time (m^2/s)

en.wikipedia.org

Molecular Diffusion

- Definition - Brownian motion, i.e. random movement of the diffusing particles

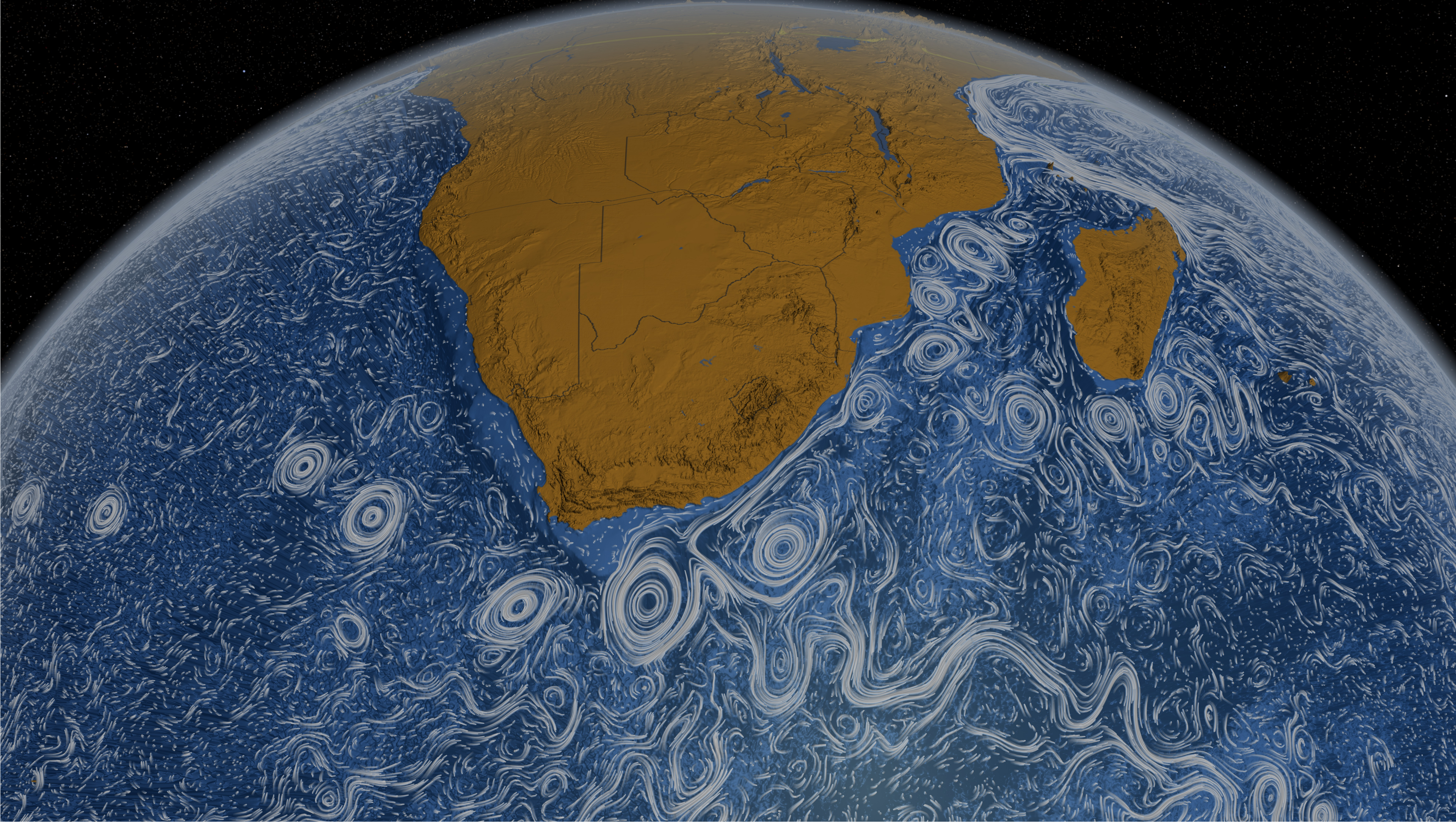
- Diffusion versus stirring

- Diffusive time scale?

Cream in coffee cup versus dye in beaker

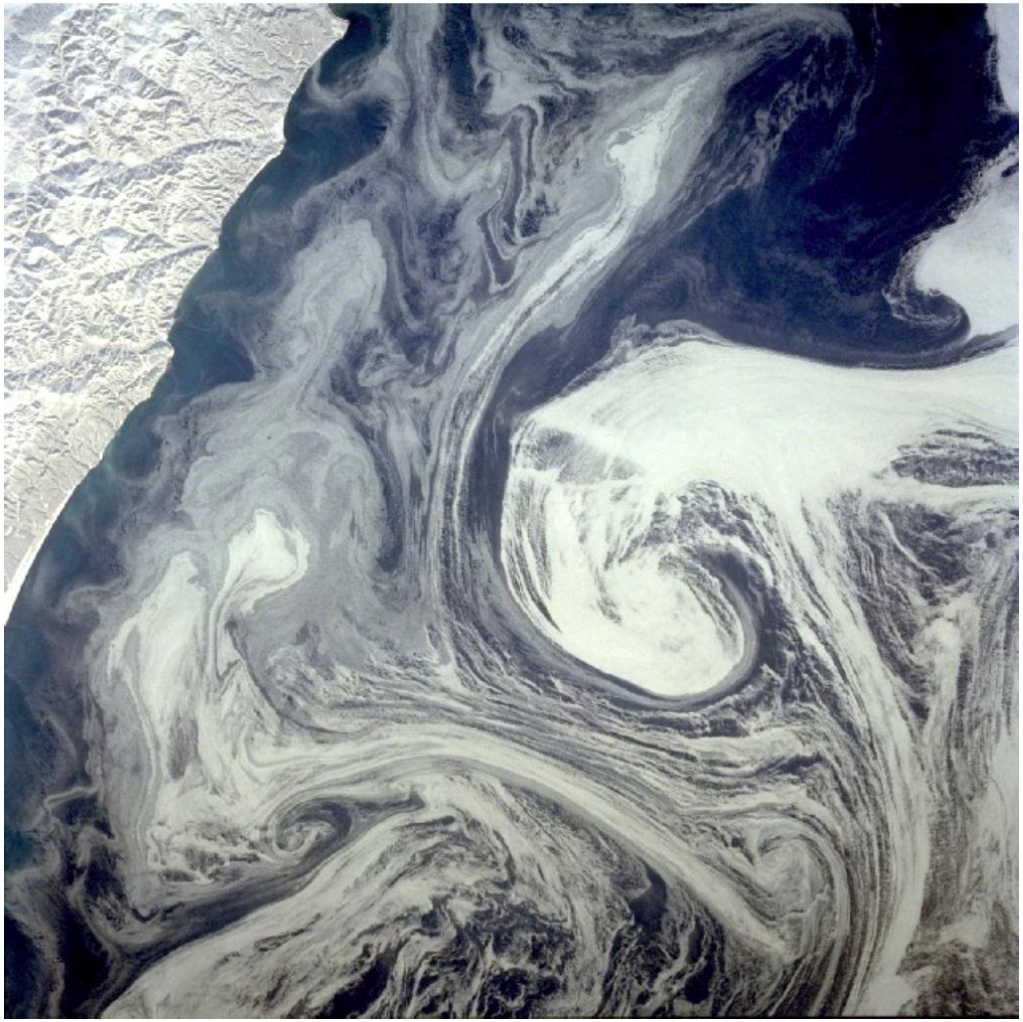
Assume that the diffusivity coefficient for the dye in water is $10^{-9}m^2/s$ and the beaker diameter is 15 cm.

How long does it take for the dye to diffuse thru the beaker under pure molecular diffusion?

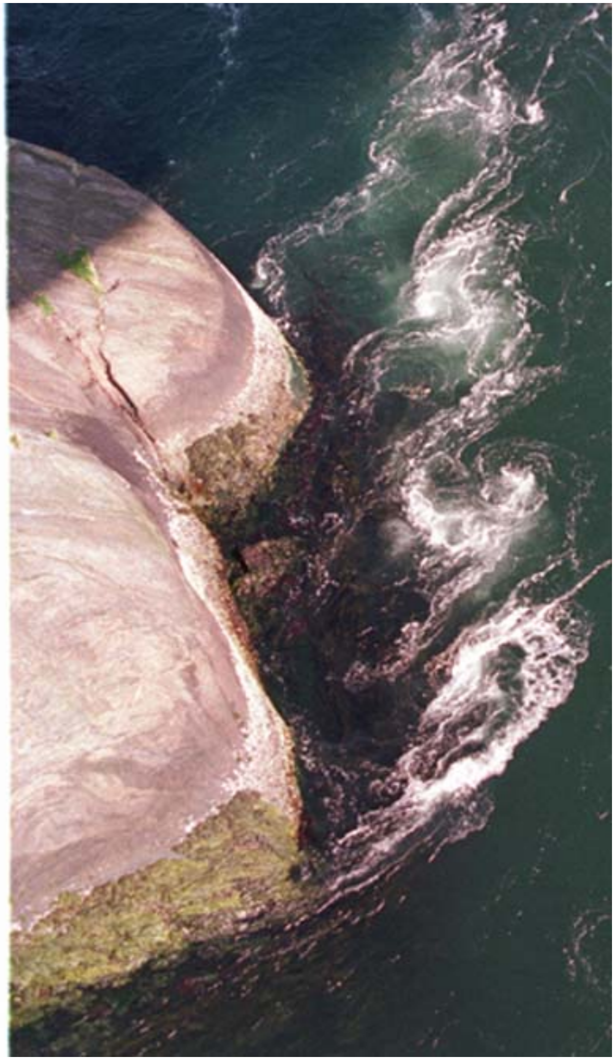


NASA Scientific visualization studio:

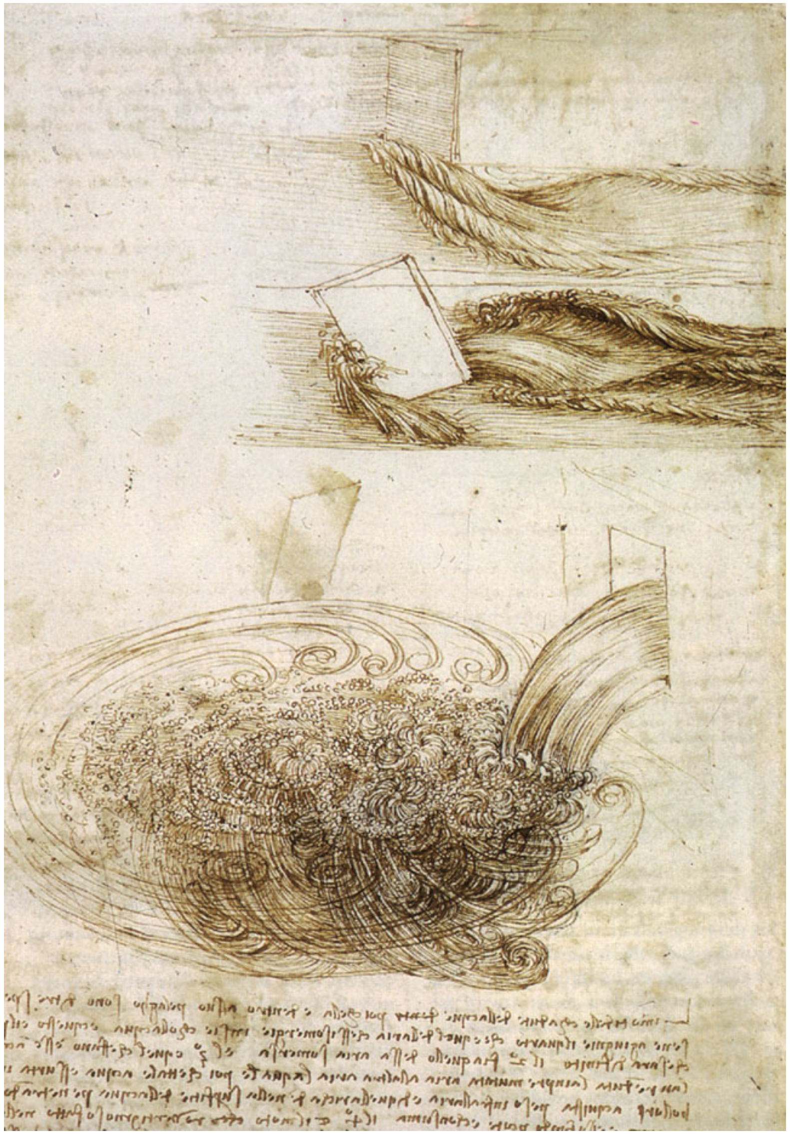
The perpetual ocean – currents and eddies near South Africa

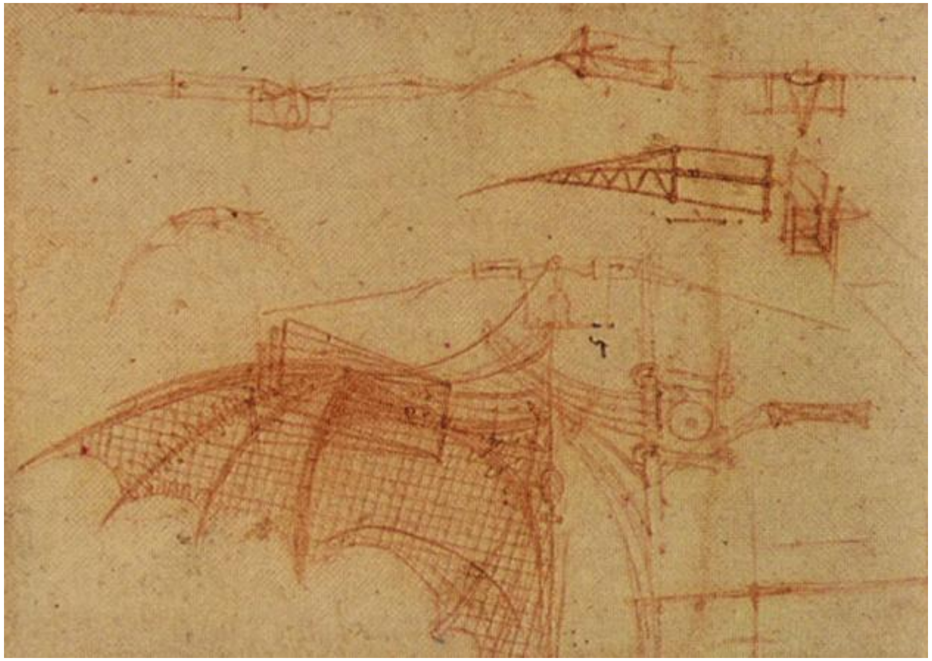


EDDIES IN THE OYASHIO CURRENT
G. K. Vallis
Princeton University
The photograph from NASA shows eddies in the Oyashio Current, off the Kamchatka Peninsula in the Bering Sea, in March, 1992. The current, part of the the western boundary current of the North Pacific subpolar gyre, is baroclinically unstable and sea-ice provides flow visualization.



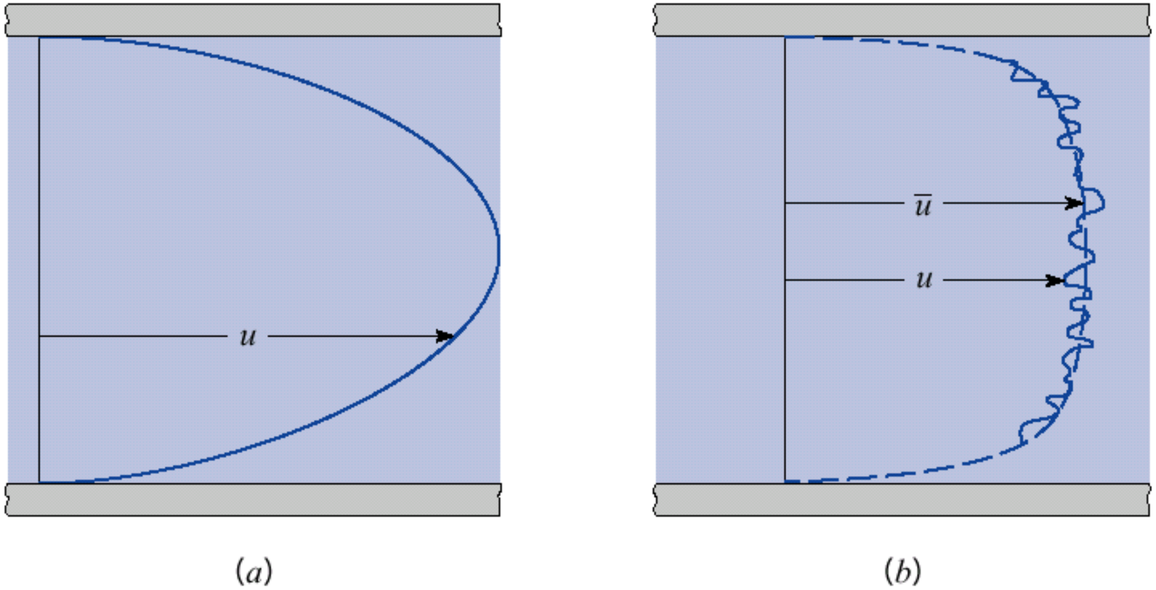
Separation off Deception Pass, WA





Laminar and Turbulent flows

- Example below – velocity profile in a straight pipe. Which one is the turbulent case?



- Determined by the value/range of the Reynolds number

Reynolds number

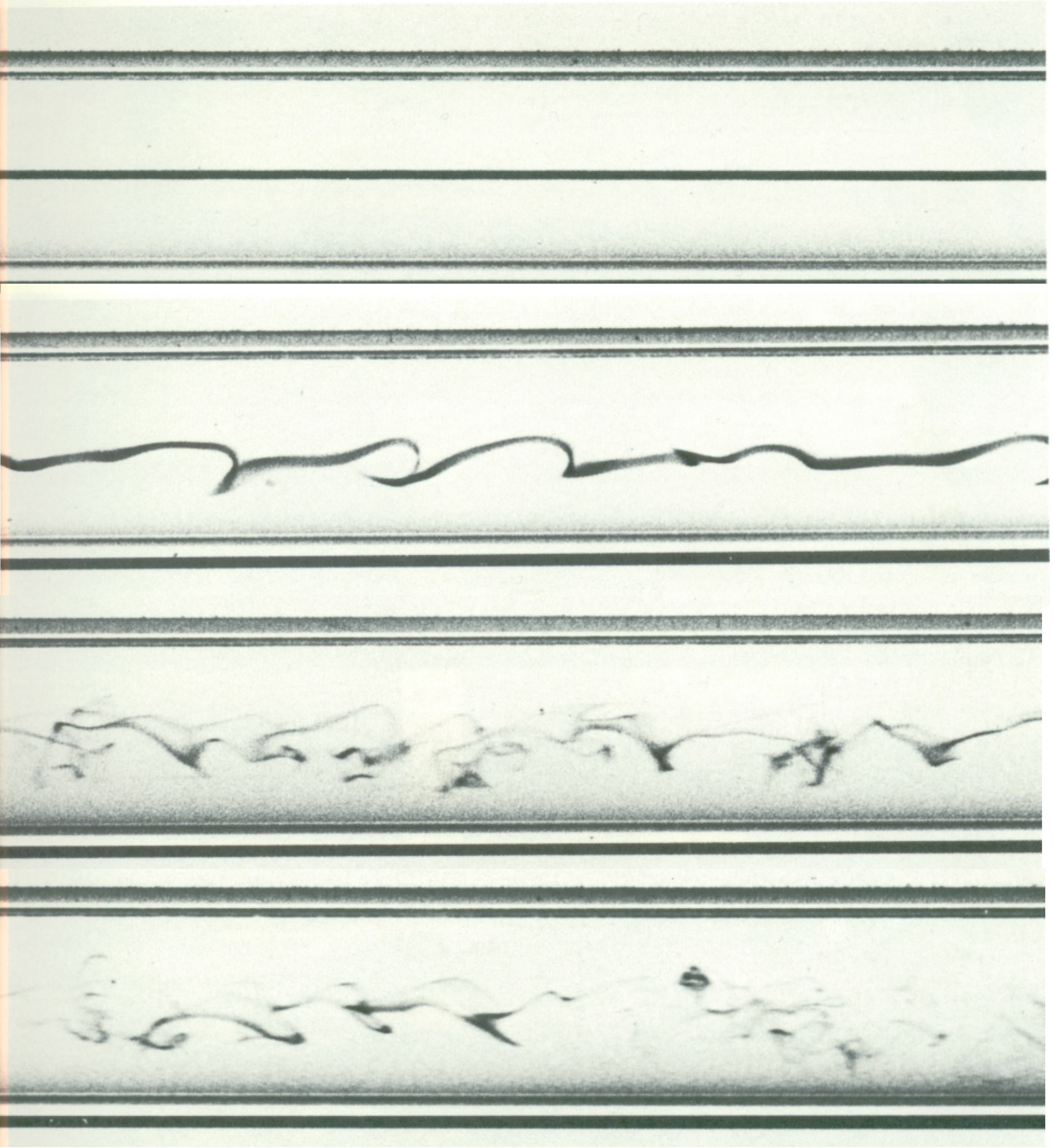
$$\mathrm{Re} = \frac{UL}{\nu}$$

- U is the flow velocity (m/s), L the relevant length scale (m) and ν the kinematic viscosity (m^2/s). ν=μ/ρ, where μ is the dynamics viscosity and ρ the density.

- Determines the transition from laminar to turbulent flows, approx. 2000 to 3000 for pipe flows for example.

- Important in determining turbulent eddy spectrum in terms of size, velocity and rotation period. Important in predicting mixing in industrial and environmental flows.

Reynolds' experiment



Laminar

Transitional

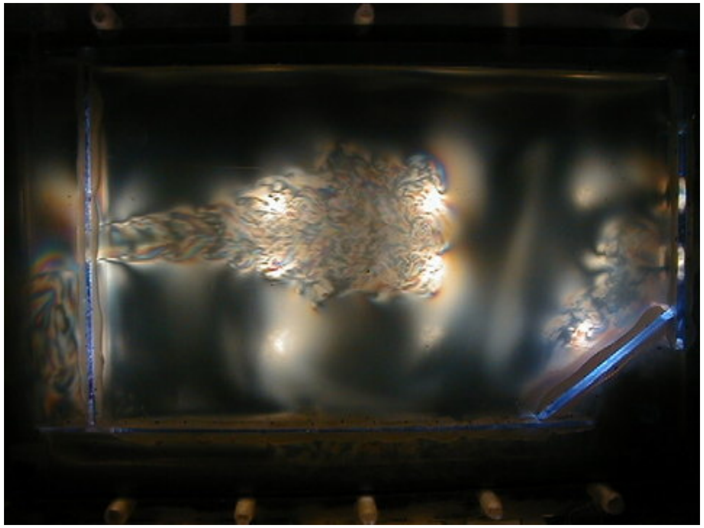
Turbulent

Fully turbulent

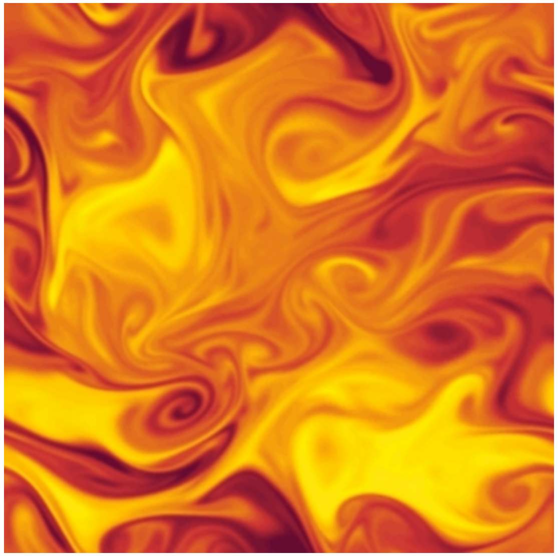
Re

Understanding turbulent flows

- At least 2 ways:
  1. Through its physical properties – using dimensional analysis, flow visualization and intuition
  2. Mathematical definition – Necessary for theory and numerical modeling



Jet in ambient fluid, V=2cm/s, 15cm horizontal field of view



Isotropic turbulence – Sc=25

Properties of turbulence

- Flows are described by the Navier-Stokes equations and corresponding boundary conditions. Continuum approximation and Newtonian fluid.
- One of the last unresolved problems of classical physics.
- Describe all the complexity of fluid flows.

$$\frac{D\vec{u}}{Dt} + 2\vec{\Omega} \times \vec{u} = -\frac{1}{\rho_o}\nabla p + \frac{\rho}{\rho_o}\vec{g} + \vec{F}$$

Gravity: pressure gradient and buoyancy

Friction: wind stress

Coriolis Force

Millennium Prize

Win a million dollars with maths, No. 3: The Navier-Stokes equations!!!

These equations were developed simultaneously in the early 1800s by George Stokes in England and Claude-Louis Navier in France. But if they have been around so long, what's the million-dollar prize for?

The problem is that for most situations the Navier- Stokes equations are too hard to solve; they tend to result in partial differential equations that are simply too complicated. For the most part, mathematicians have developed numerical workarounds to squeeze out solutions to the equations, but it's a dark art.

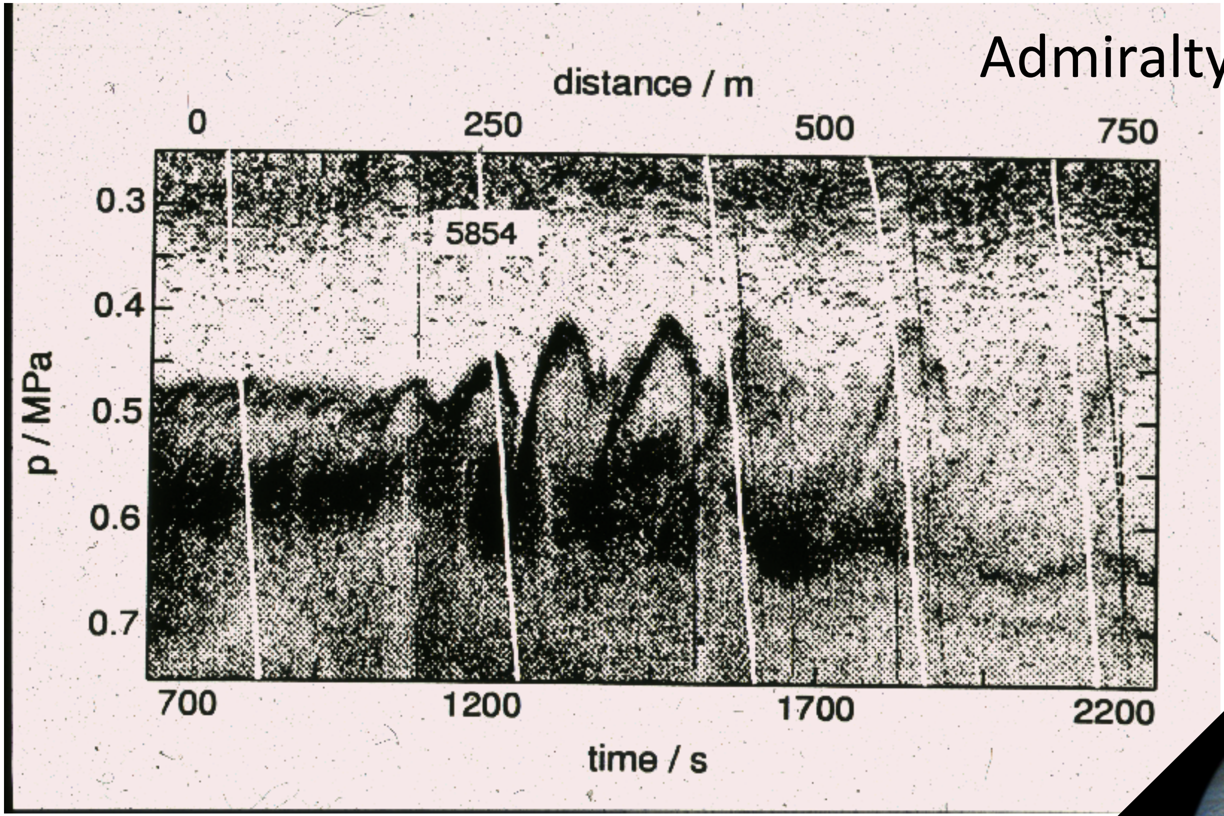
http://www.guardian.co.uk/science/blog/2010/dec/14/million-dollars-maths-navier-stokes/

Other characteristics of turbulent flows

- Turbulent flows are highly irregular. Chaotic, random but not completely. Expected to have some structures: eddies!

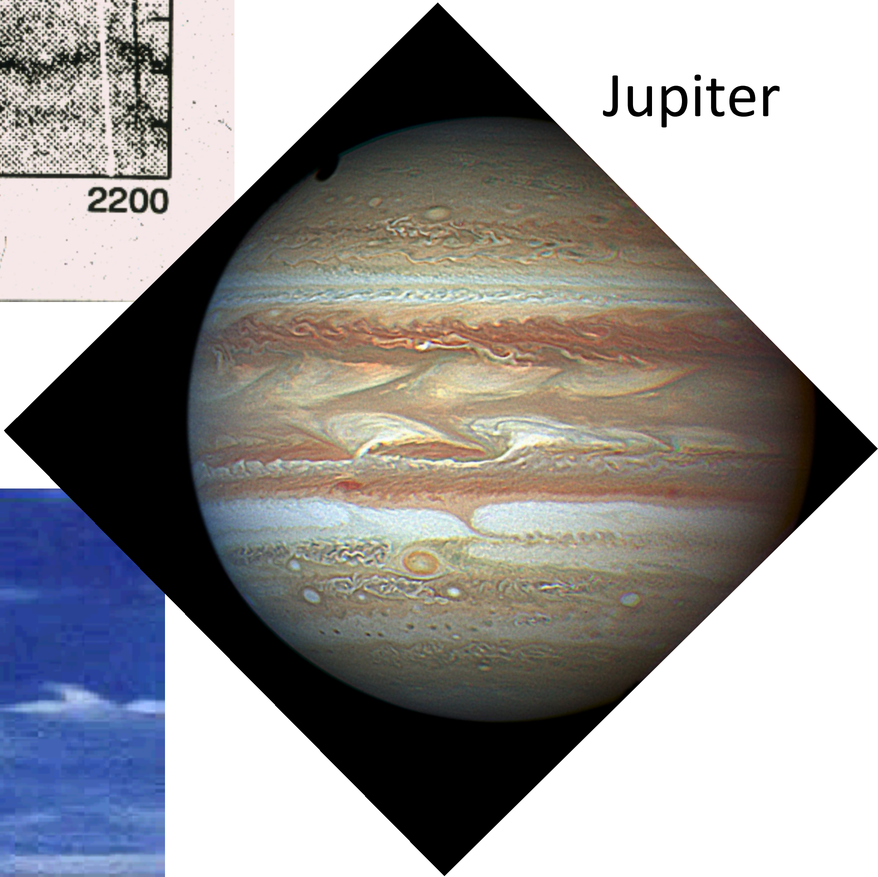
- Turbulent flows are unstable. They are the results of flow instabilities. Remember Reynolds' experiment.

- Strong sensitivity to initial and boundary conditions.



Admiralty inlet, WA – shear layer

Jupiter

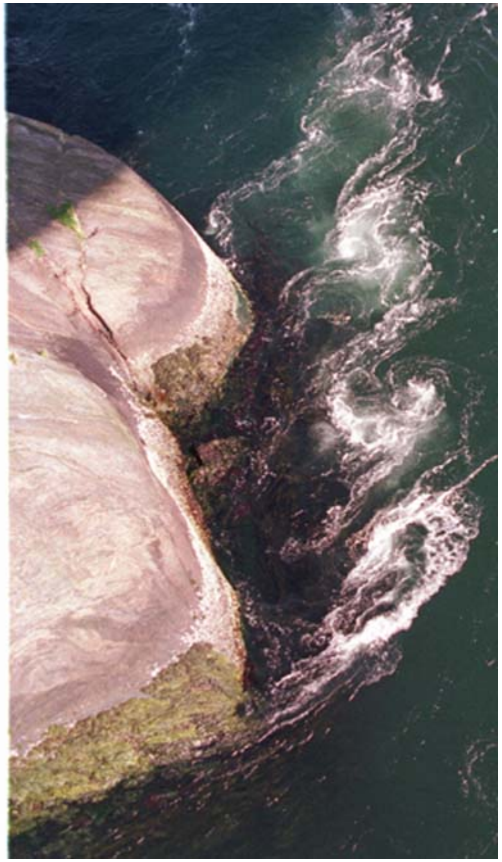


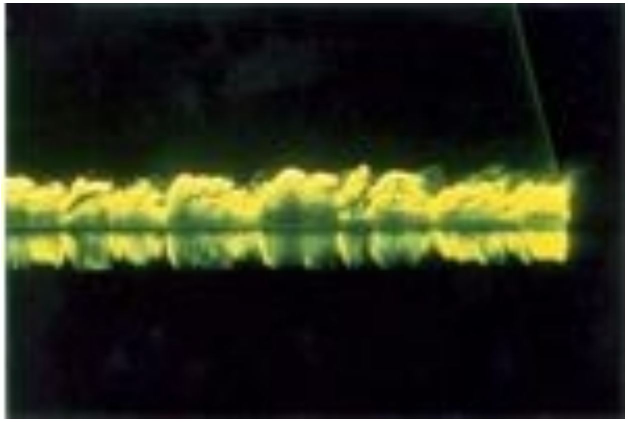
Shear layers in the atmosphere

Other characteristics of turbulent flows

- Turbulent flows are strongly non linear making it hard to solve the Navier-Stokes equations.

- Turbulent flows are highly vortical, i.e. characterized by a large amount of vorticity (similar to angular velocity).





Large eddies in a turbulent boundary layer

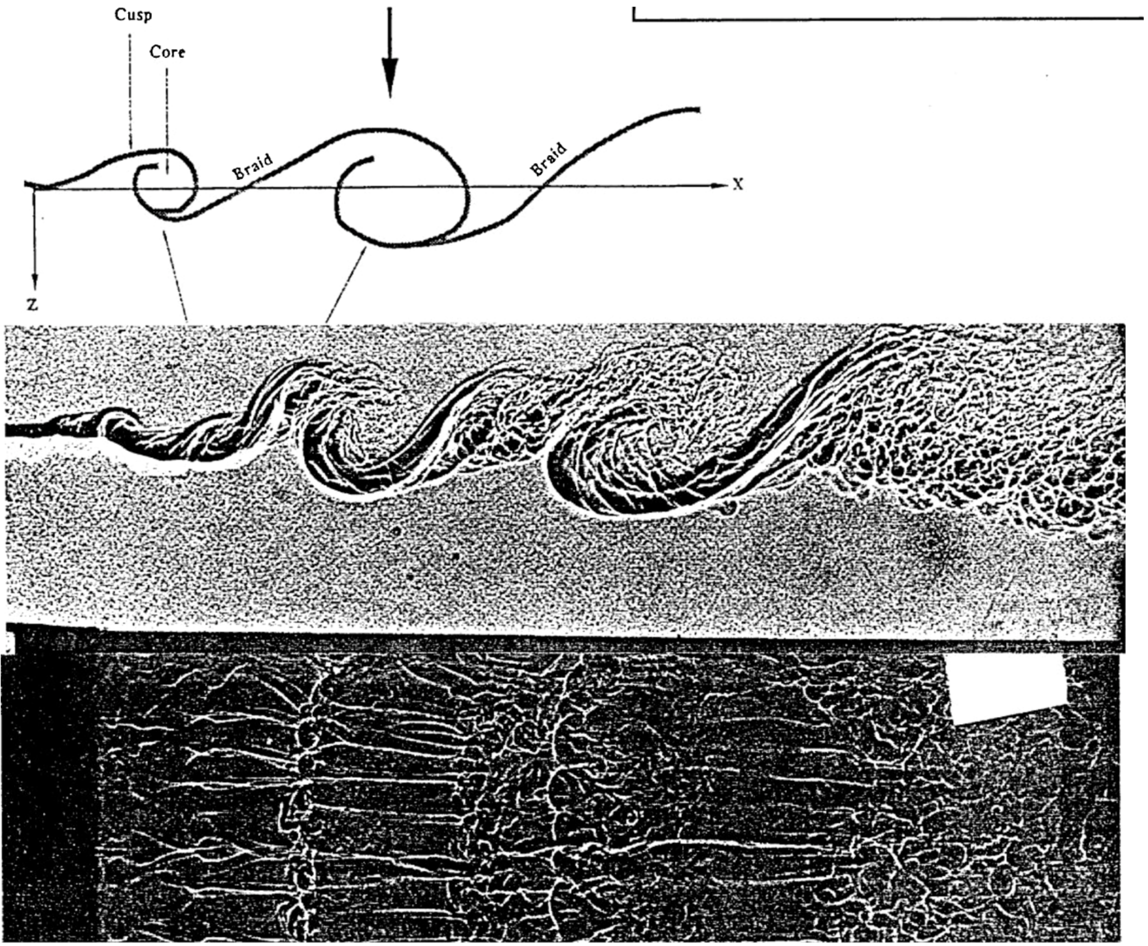
Characteristics cont'd

- Turbulent flows are 3-dimensional.
- Vortex tube stretching is a key process -> it increases vorticity in the stretching direction (because of the conservation of angular momentum).

http://www.smithsonianchannel.com/videos/mesmerizing-pufferfish-twirling-in-bubble-lassos/37447

- Turbulent flows are highly dissipative. Mechanical energy is transformed into internal energy. Need a supply of energy to sustain it.
- For the ocean, the source of energy can be mechanical and/or convective.

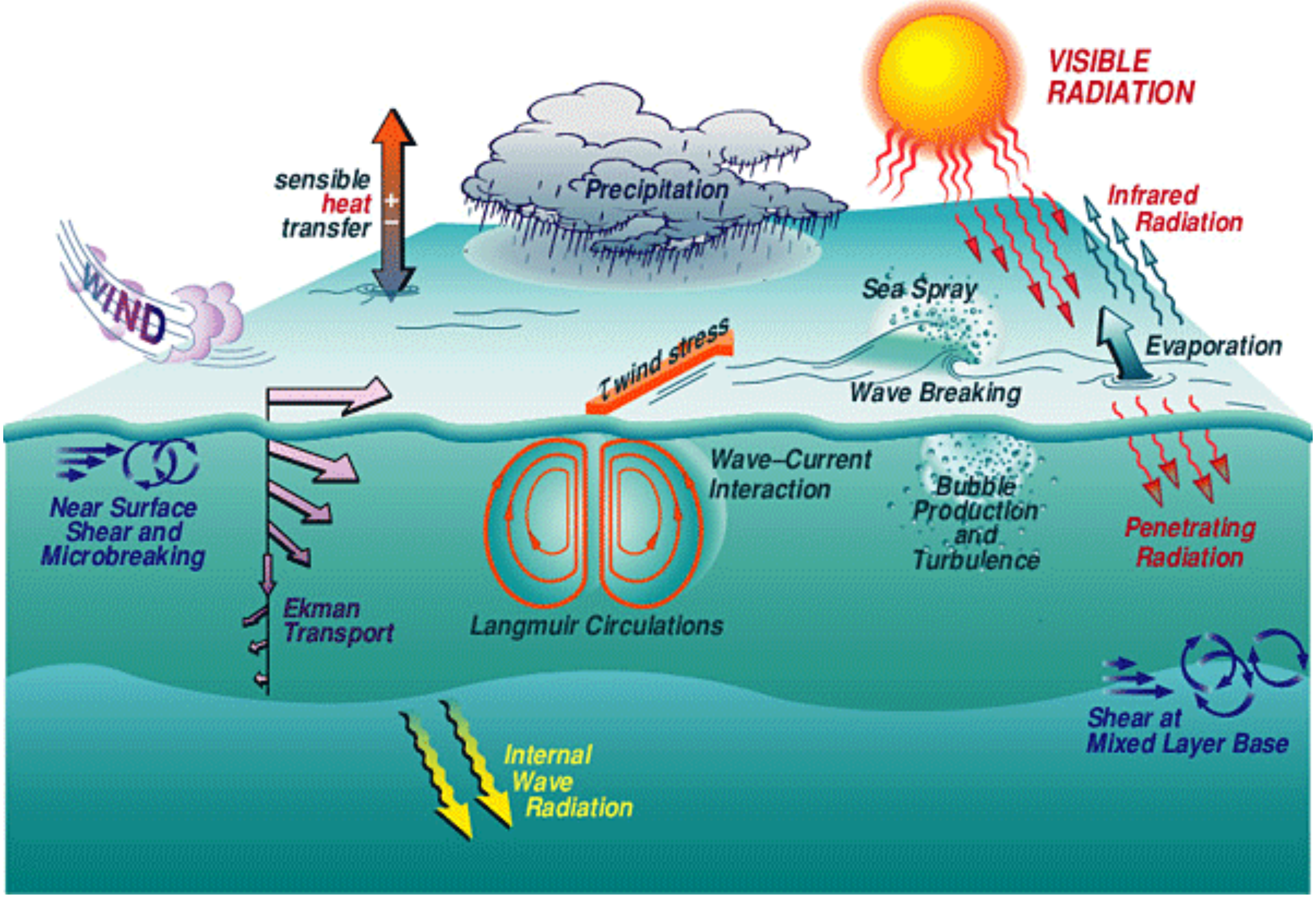
Kolmogorov's energy cascade theory (1941)



Relationship between the largest eddies in the flow and the smallest ones where diffusion becomes dominant

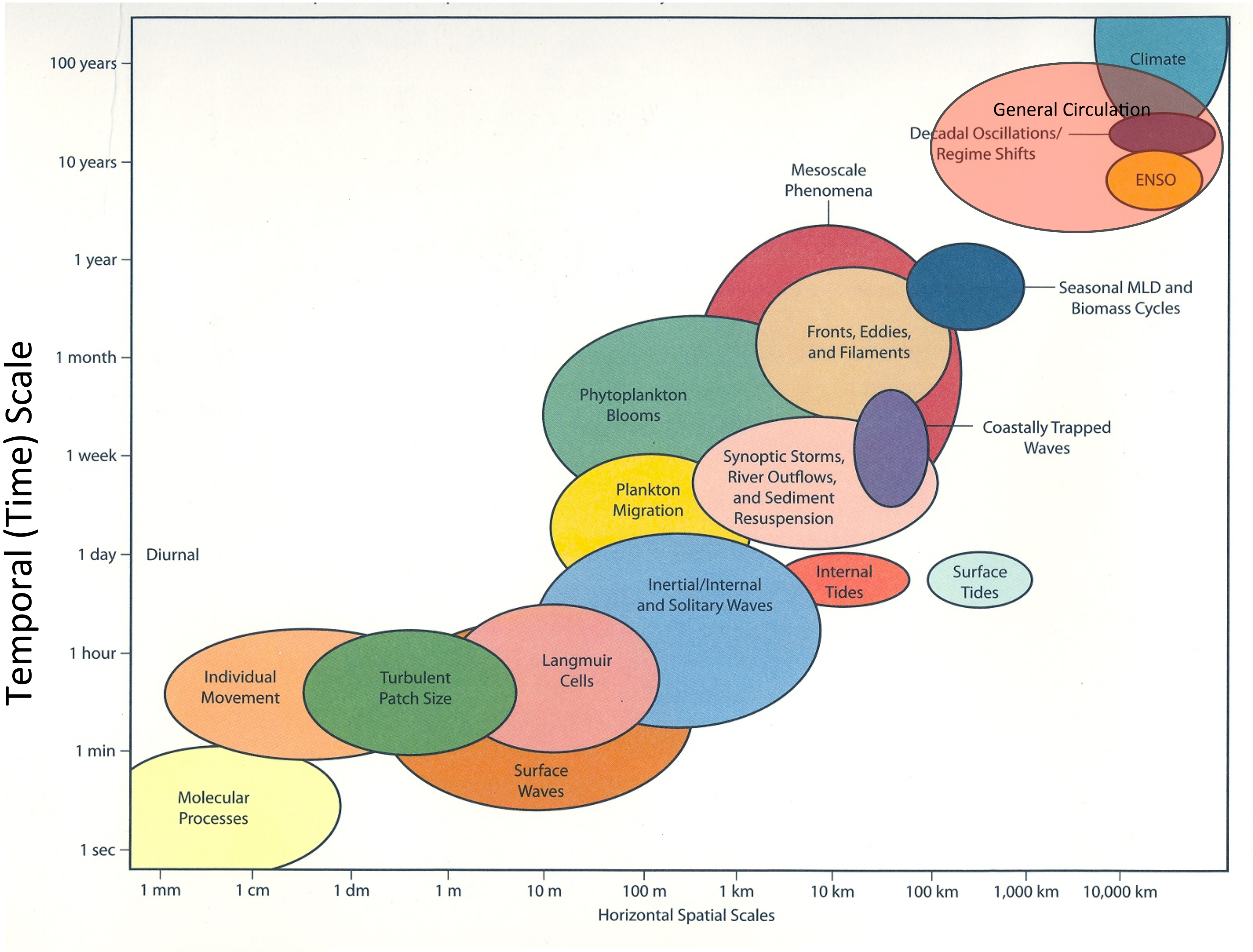
This relationship is a function of Re

Important for the estimation of spatial and temporal resolutions of ocean instruments



Schematic drawing of processes that contribute to variability in the ocean surface boundary layer.
[R. Weller, WHOI]

Temporal and spatial scales of various physical ocean processes



Temporal (Time) Scale

Horizontal Spatial Scale

Image credit: Exploring the World Ocean, inside cover

Physical approach - using dimensionless parameters

- Reynolds number: $\mathrm{Re} = \frac{UL}{\nu}$
- Richardson number: $Ri = \frac{(\Delta\rho/\rho)gL}{U^2}$
- Rossby number: $Ro = \frac{U}{Lf}$

where f is the Coriolis frequency, $f = 2\Omega \sin\theta$
Ω: planet rotation
Θ: latitude

No stratification

With stratification

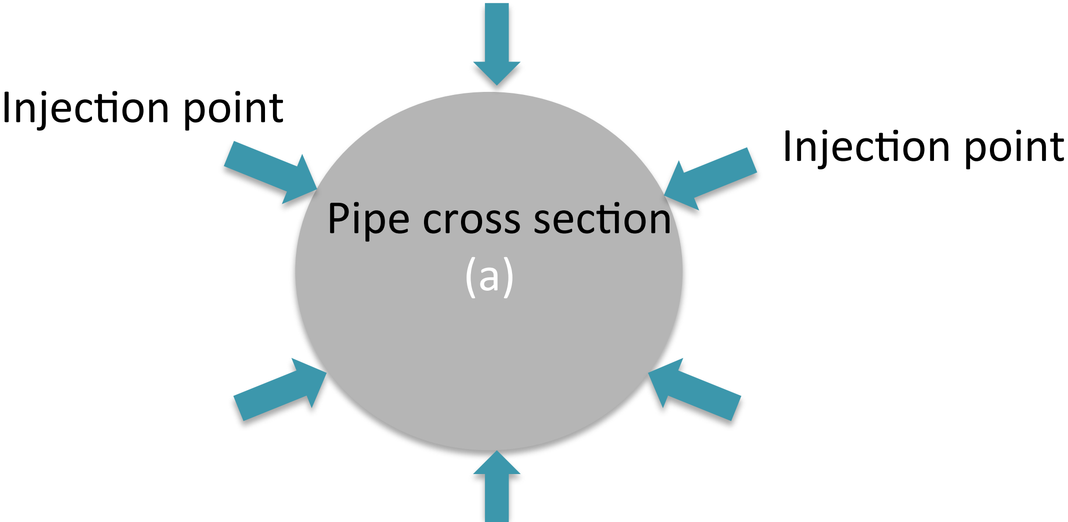
The entrainment and mixing can be modeled based on dimensionless parameters such as Re and Ri.

Summary

- Explored the concept of diffusion
- Compared it to turbulence
- Looked into the properties and characteristics of turbulent flows
- Will apply these ideas/concepts in Thursday's lab.

What’s the most efficient configuration?

- Mixing 2 fluids thru injection.
- Choice of configuration:
  - Option 1 : (a) is better
  - Option 2: (b) is better
  - Option 3: There is no difference

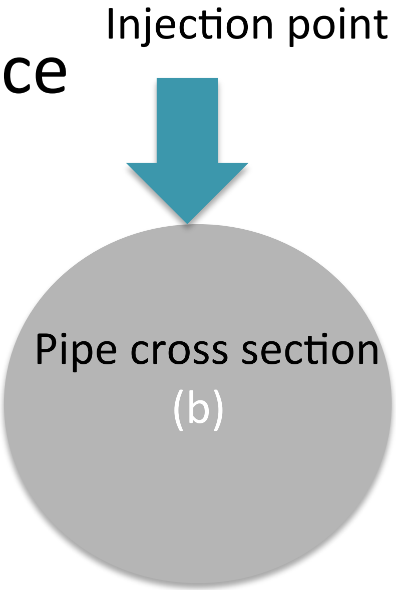


Injection point

Injection point

Pipe cross section

(a)



Injection point

Pipe cross section

(b)