# Intro to dimensional analysis

THURSDAY LAB#3

## 1. Diffusion

a) It is desired to evaluate changes in salt concentration (salinity) in a quiescent tank of water 10 m deep in which a salinity gradient has been established in the vertical direction. The tank is not stirred and is completely sealed from its surroundings (no water or salt is added or taken away). Estimate the time required for the gradient to be smoothed by molecular diffusion (molecular diffusivity for salt in water is $10^{-9}$ cm²/s). Does the magnitude of the initial gradient make any difference?

Pure diffusion process so

$$L = \sqrt{Dt}$$

The time it takes is then $10^{11}$s. No effect of initial gradient.

b) What if we are looking at heat diffusion thru the same tank? The thermal diffusivity for water is $10^{-7}$ m²/s. How long does it take now?

c) If turbulence is present, what will be the time for the gradient to be smoothed out? Assume a turbulent diffusivity of $10^{-5}$m²/s.

## 2. Diffusion versus turbulence

Qualitatively show the radial profiles of velocity, temperature, and concentration for an axisymmetric heated jet of fluid A (*sewage discharge in a well mixed bay for example*), introduced into quiescent ambient fluid B.  Assume D > α > ν, where D and α are mass and thermal diffusion coefficients, respectively, and ν is the kinematic viscosity.  Make 2 separate plots at x/d = 0.5 and x/d = 30.

At $\frac{x}{d}$ = 0.5 , shear layer has not rolled up yet -

At that location , mixing demunded by diffusion -

At $\frac{x}{d}$ = 30 , mixing controlled by large scale eddies -  little effect from diffusion

$\bar{c}$
$\bar{T}$
$\bar{v}$

$\frac{x}{d} = 0.5$

$\bar{c}, \bar{T}, \bar{v}$

$\frac{x}{d} = 30$
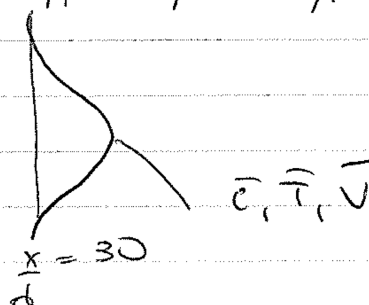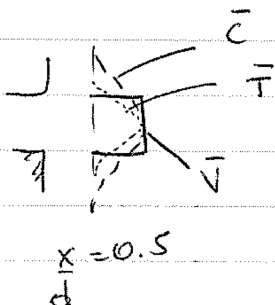
## 3. Momentum versus buoyancy

Using a beaker, we will create a vertical jet (with a syringe) and a vertical plume (different density fluid) and look at the difference in spatial spreading and speed in both cases. Alos notice the difference in eddy structure as the flow moves downstream.

Demonstrate thru dimensional analysis the relationship between width/spread and centerline velocity as a function of time and downstream distance for jets versus plumes.
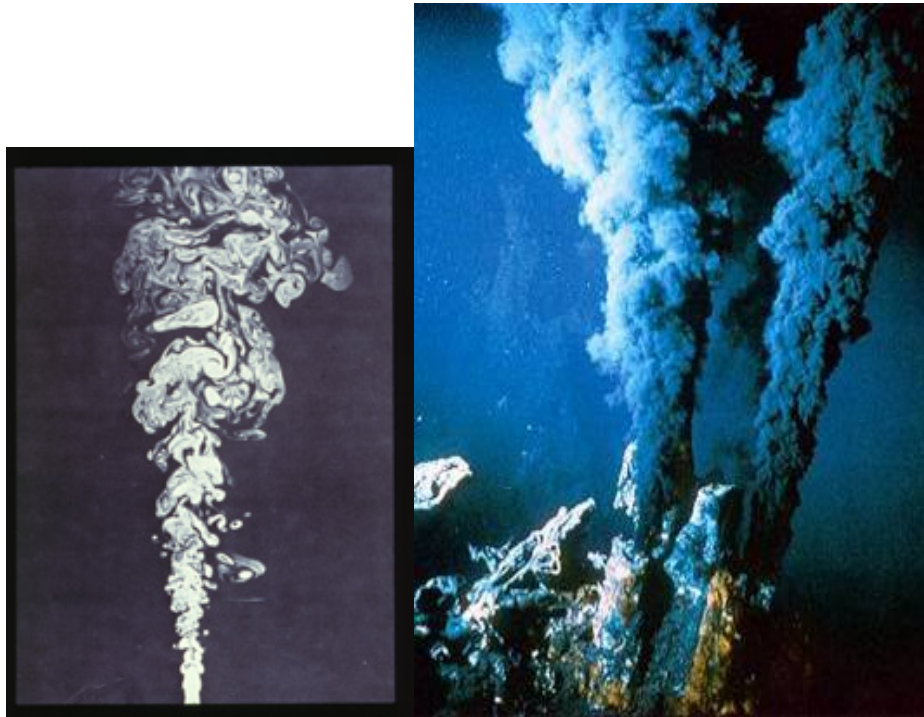


Album of Fluid Motion, Van Dyke    oceanography101.wikispaces.com

## 3-D JETS

$$T = \rho U^2 \delta^2$$

$\delta \propto x$   No other length scale

$$T = \rho U^2 \delta^2 = \rho \frac{\delta^2}{t^2} \delta^2$$

$$\frac{T}{\rho} t^2 = \delta^4 \longrightarrow \boxed{\delta \sim \left(\frac{T}{\rho}\right)^{1/4} t^{1/2}}$$

$$U \sim \frac{\delta}{t} \sim \boxed{\left(\frac{T}{\rho}\right)^{1/4} t^{-1/2}}$$

$$T = \rho U^2 x^2$$

$$U^2 = \frac{T}{\rho} \frac{1}{x^2} \longrightarrow \boxed{U \sim \left(\frac{T}{\rho}\right)^{1/2} \frac{1}{x}}$$

Rotation rate: $\dfrac{U}{\delta} \sim \dfrac{\left(\frac{T}{\rho}\right)^{1/4} t^{-1/2}}{\left(\frac{T}{\rho}\right)^{1/4} t^{1/2}} \sim \dfrac{1}{t}$

## Plume

$$F = 2\pi \int_0^\delta \rho' w \, r \, dr \qquad \text{Buoyancy flux is conserved}$$

$$= \frac{\pi d^2}{4} \rho'_o \, w_o$$

$$d \sim z \qquad\qquad F = \rho'_o \, w \, \delta^2$$

$$\rho' = \frac{\Delta \rho}{\rho} g \qquad\qquad R_i = \frac{\rho' d}{w^2}$$

$$w^2 \sim \rho' d \qquad \rho' = \frac{w^2}{d}$$

$$F = \frac{w^2}{d} w \, d^2 = \cancel{\rho} w^3 d = \cancel{\rho} w^3 z$$

$$F \frac{1}{z} = w^3 \qquad \rightarrow \boxed{w = F^{1/3} z^{-1/3}}$$

$$\rho' = \frac{F^{2/3} z^{-2/3}}{z} = \boxed{F^{2/3} z^{-5/3}}$$