Homework 5: Tides

Assigned: November 10, 2014
Due: November 19, 2014, Before the beginning of class

Question 1) (10 points)

In this question we will explore one of the important reasons that the actual oceanic tides are not equal to the equilibrium tidal forcing.

a) Compute the speed of shallow-water surface gravity waves in water 4000 m deep. Show your work, define symbols used, and include units.

b) Compute the number of hours needed for the waves described in a) to circle all around the Earth's equator. Show your work, define symbols used, and include units.

Since the answer in b) is much larger than a lunar day, the waves do not travel fast enough to keep up with the astronomical forcing. This is one of the major reasons that the actual tide does not equal the equilibrium tide.

c) Turning the question around, how deep would the oceans have to be in order for shallow-water surface gravity waves to keep up with the astronomical forcing? Show your work, define any symbols used, and include units.

Question 2) (45 points)

In this question we will plot the equilibrium tide for both M_2 and K_1, and seek to understand some of the characteristics of these plots.

a) Run the Matlab script plot_equilibriumtide_M2.m and insert the resulting plot into your homework writeup. Note that the plot is in color; you may change it to black and white if you have to.

b) Copy plot_equilibriumtide_M2.m into a new script, plot_equilibriumtide_K1.m. Using the Topic7 lecture notes as your guide, change the values of the constants A, Lovenumbercombo, and lonsdependence, and change the array latsdependence, so that your new script plots the equilibrium K_1 tides. List your new values of A, Lovenumbercombo, and lonsdependence here. List the new line of Matlab code which you have for latsdependence. Run your new script. Attach the plot generated by your new script with this homework. Note that the plot is in color. You may change it to black and white if you have to**.**

To answer the next six questions you should examine either of the two snapshots in equilibriumtideM2.png—the .png file produced by plot_equilibriumtide_M2.m—and either of the two snapshots in your plot of the the K_1 equilibrium tide.

c) How many highs and how many lows are there in the M_2 equilibrium tide as one circles the earth along a line of constant latitude—say, for instance, the equator?

d) Is this number of highs and lows consistent with the factor of "2λ" in the argument of the $\cos(\omega t+2\lambda)$ term in the M_2 equilibrium tidal forcing?

e) How many highs and how many lows are there in the K_1 equilibrium tide as one circles the earth along a line of constant latitude—say, for instance, along 50°N?

f) Can you explain your answer in e)? Hint: follow the same reasoning as you did in d).

For the next two questions, in addition to examining your plots, you will find it helpful to consider the latitudinal dependence in the formulae for equilibrium tidal forcing, given in the lecture notes. In answering h) you might find it helpful to also examine the slides on "Declination" in the Topic 7 notes.

g) Is the M_2 equilibrium tide symmetric around the equator, or anti-symmetric? A symmetric function G is one in which $G(-\phi)=G(\phi)$, where ϕ is latitude. An anti-symmetric function G is one in which $G(-\phi)=-G(\phi)$.

h) Is the K_1 equilibrium tide symmetric around the equator, or anti-symmetric? Can you explain why this should be so?

Question 3) (45 points)

In this question we will examine the actual M_2 and K_1 surface tidal elevations—in other words, the response to the equilibrium tidal forcing discussed in question 2.

Begin by examining the plots m2ampandphase.pdf, and k1ampandphase.pdf, included in the homework package. These plots were produced in Matlab, from the GOT99 tide model (Richard Ray, 1999, NASA Technical Report). GOT99 is a tide model derived from satellite altimeter data. Note the locations of large tides in the amplitude plots, and the sense of rotation in the phase plots.

Next run the script plot_snapshots_m2actualtide.m in Matlab. This script plots global snapshots of the actual M_2 surface tidal elevations, 180 degrees apart. As you can see from examining the script, the snapshots are computed from the amplitude and phase of M_2, which were plotted above. Note the correspondence of the highest highs and lowest lows in the snapshots with the regions of large amplitude in the plots examined above.

Next run the script plot_snapshots_k1actualtide.m in Matlab, and note once again the correspondence of the highest highs and lowest lows in the snapshots with the regions of large amplitude in the plots examined above.

Note the very substantial differences between the snapshots of actual tidal surface elevations, and the equilibrium elevations. One of the main reasons for the differences was examined in question 1 above. Additional reasons are given in the lecture notes.

a) Compare the snapshots of actual M_2 surface tidal elevation versus the equilibrium tide for M_2. Which do you think would be larger, in a global average? Hint: consider the difference in the axis limits for the two plots. Keep in mind that this is not a trick question. If you want to make the answer even more clear to yourself, rewrite and rerun the plot_equilibrium… and plot_snapshots… codes to force the same axis limits for both plots. See the lines "set(gca,'Clim',[0 1])" etc. If you don't trust even that, compare Tables 1 and 2 in the paper "dsr2tidesarbicetal.pdf" included in this homework, and keep in mind that the equilibrium amplitude A is multiplied by three things—the Love number combination $1+k_2-h_2$, the latitudinal dependence $\cos^2(\phi)$, and the longitudinal and time dependence $\cos(\omega t+2\lambda)$—all of which are less than one.

The difference between the actual and equilibrium M_2 surface tidal elevations is due to resonance. The K_1 tide is also resonant, but in general, not to the same degree that M_2 is.

b) Compare the amplitude plots of the actual tidal elevations, for M_2 versus K_1. Once again keep in mind the difference in scales. Which of the two constituents do you think would be larger in the global average? Once again this is not a trick question. You may look at Table 2 in the paper "dsr2tidesarbicetal.pdf" included in this homework, if you want confirmation of your instincts.

c) Compare the snapshots of the equilibrium tides, for M_2 versus K_1. Which do you think would be larger in the global average? Again, not a trick question.

Next we will explore whether there are any particular locations where the actual K_1 tide exceeds the actual M_2 tide.

From the Trujillo and Thurman plot on "Tidal types" in the lecture notes, we see that "Pakhoi, China" is listed as having a diurnal tide.

"Beihai" is another name for "Pakhoi". From the Wikipedia article on "Beihai, China" we see that this city has a latitude of about 21.5°N, 109°E. Type the following into Matlab (note the "index=.." are Matlab's responses to your typing, you don't type these in):

```
>>load GOT99_M2andK1.mat
```

```
>> index=find(lats==21.5)

index =

  224

>> index=find(lons==109)

index =

  219
```

Thus typing out

```
>>ampk1(224,219)
```

followed by

```
>>ampm2(224,219)
```

will yield the actual K_1 and M_2 tidal amplitudes in meters at Pakhoi, China.

d) What is the actual K_1 tidal amplitude in meters at Pakhoi, China?

e) What is the actual M_2 tidal amplitude in meters at Pakhoi, China?

f) Is the Trujillo and Thurman plot referred to above correct in its assessment that the Pakhoi tide is predominantly diurnal?

g) Following the procedure above, find the amplitude of the actual K_1 tide in meters for the Sea of Okhotsk (latitude~=61°N, longitude~=163.5°E).

This is the location of the world's largest diurnal tides.

h) Find the amplitude of the actual M_2 tide at this same location.

Use the same procedure to find the amplitude of the actual tide in meters, in some regions famous for having large semidiurnal tides. List both the M_2 and K_1 tidal amplitudes, in meters, for the remaining questions. Remember that latitudes south of the equator are negative in the Matlab GOT99 data.

i) Hudson Strait (59.5°N, 291°E)

j) English Channel (49°N, 358°E)

k) Bay of Fundy (45°N, 294°E)

l) Patagonian Shelf (51°S, 291°E)

Note that many of the locations of large M_2 tides in the world ocean occur far from the equator, where the forcing is a maximum. The maximum response does not have to occur where the forcing is maximum because of the **non-locality** of the tidal problem—tidal solutions at any location are affected by the solutions elsewhere, which propagate rapidly around ocean basins.

Question 4) (20 points EXTRA CREDIT) (HIGHER-LEVEL MATH)

a) Following the derivation in the lecture notes for the quarter-wave resonance on a shelf, derive the half-wavelength resonance condition $L=n\lambda/2$, where n is an integer and λ is wavelength, for a basin of depth H and length L which is closed at both ends.

The half-wavelength resonance condition can also be written as $\omega L/\sqrt{gH} = n\pi$ for integer n.

b) For H=4000 m, ω=the M_2 frequency, and n=1, what value of L is resonant? Is this scale comparable to ocean basin scales?

c) For H=4000 m, ω=the K_1 frequency, and n=1, what value of L is resonant? Is this scale comparable to ocean basin scales?

It is sometimes stated that the differences between the answers in b) and c) are consistent with the fact that the Pacific has larger diurnal tides than the Atlantic. The larger basin resonates more easily with lower frequencies than the smaller basin.