Tides

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Outline

- Equilibrium tide, lunar vs. solar vs. sidereal days, spring-neap cycle, declination and diurnal tides
- Harmonic decomposition
- Tidal species
- Laplace's tidal equations
- Tidal response to astronomical forcing
- Coastal tides

Motivation

• Although the study of tides dates back to Newton, they remain an important part of physical oceanography.

• Satellite altimetry has revolutionized our understanding of tides by producing amongst other things accurate global maps of tidal elevations.

• Tides are an important signal in current meter data, tide gauge records of sea level, and other instruments.

• They are important for navigation.

• Tides are an important source of mixing in both the coastal and open-ocean.

Equilibrium tide I: Cause

• Tidal forces are due to the differential force of gravity across a body of finite size–for instance the Earth rotating around the center of mass of the Earth-Moon system.

• The next five slides give a cartoon view, from a lower-level oceanography course (Essentials of Oceanography, Trujillo and Thurman)

Earth-Moon system rotation

- Barycenter between Moon and Earth revolves around Sun

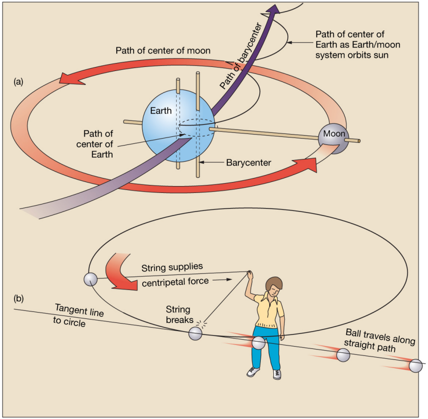

Fig. 9.1, page 278

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Gravitational forces

- The gravitational force is largest on the side of the Earth closer to the moon and least on the side further from the Moon. The force is always directed toward the center of the Moon.

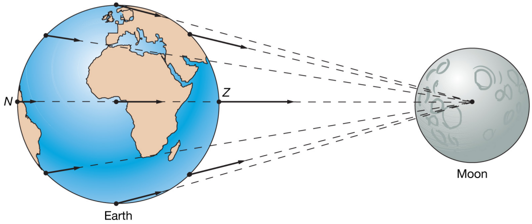

Fig. 9.2, p. 279

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Centripetal force

- Due to rotation about “barycenter” or center of mass between two bodies in orbital motion.
- The centripetal force (the red arrows) is everywhere the same. The red arrows are all the same length and point in in the same direction.

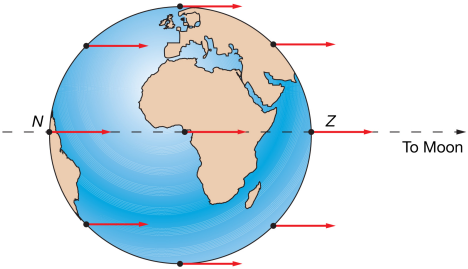

Fig. 9.3, p. 279

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Tide-producing (resultant) forces

- Resultant forces = differences between centripetal and gravitational forces
- The **tidal generating force** (blue) is the **difference** of the **Moon's gravitational force** (black) and the **centripetal force** (red). Tide-generating forces point towards the Moon at Z (zenith) and away from the Moon at N (nadir).

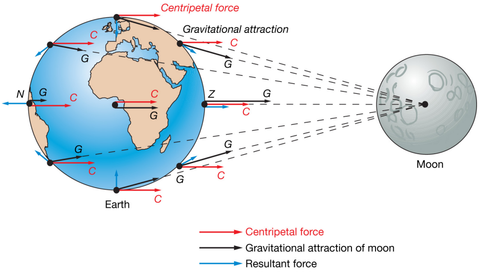

Fig. 9.4, p. 280

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Equilibrium tide, Trujillo and Thurman

Tidal bulges (lunar): Ideal Earth covered by ocean

- One bulge faces Moon
- Other bulge on opposite side
- Tidal cycle results from earth rotating underneath these bulges→ two high tides and two low tides during a day
- The simple shape in the figure is called the equilibrium tide.
- Actual tide in oceans is a complex response to forcing of equilibrium tide

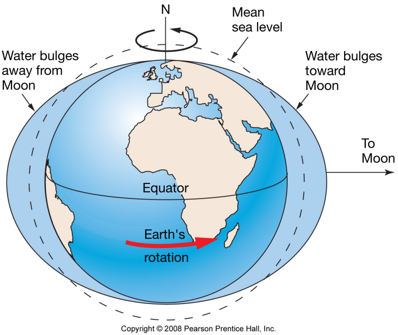

Fig. 9.6, p. 281

Equilibrium tide II: Mathematics

• All points on Earth experience the same centripetal force per unit mass due to motion around the center of mass of the Earth-Moon system:

$$F_{centripetal} = \frac{GM_{moon}}{r^2},$$

where G is Newton's gravitational constant, M_{moon} is the mass of the moon, and r is the distance from the center of the moon to the center of the Earth.

• On side of Earth closest to Moon, the gravitational pull of the Moon is given by

$$F_{gravitational} = \frac{GM_{moon}}{(r-a)^2},$$

where a is the radius of the Earth.

Equilibrium tide mathematics continued

$$Tidal\ force = F_{gravitational} - F_{centripetal} =$$

$$GM_{moon}[\frac{1}{(r-a)^2} - \frac{1}{r^2}] \approx$$

$$\frac{GM_{moon}}{r^2}[\frac{1}{1-\frac{2a}{r}} - 1] \approx \frac{2aGM_{moon}}{r^3}$$

- On side of Earth farthest Moon, we have

$$Tidal\ force = F_{gravitational} - F_{centripetal} =$$

$$GM_{moon}[\frac{1}{(r+a)^2} - \frac{1}{r^2}] \approx \frac{GM_{moon}}{r^2}[\frac{1}{1+\frac{2a}{r}} - 1] \approx \frac{-2aGM_{moon}}{r^3}$$

- The force is equal but opposite, yielding two bulges in the equilibrium tide.

Equilibrium tide III: Solar vs Lunar Tides

- The same reasoning applies to the Sun's gravity field on the earth.
- Therefore the sun should also cause tides.
- Show of hands: which are larger?
  - solar tides
  - lunar tides

Solar vs Lunar tides continued

$M_{sun} = 27{,}000{,}000 \ M_{moon}$
$r_{sun} = 390 \ r_{moon}$
$\frac{M_{sun}}{r_{sun}^3} = 0.45 \frac{M_{moon}}{r_{moon}^3}$

• Sun loses to Moon when considering differential gravity (cause of tides).

Period of principal solar vs principal lunar tides

- Is the period of the principal lunar tide equal to that of the principal solar tide?

- Show of hands
  - equal
  - not equal

Lunar day versus solar day (Trujillo and Thurman)

Key question—how is a lunar day different from a solar day? (p. 281)

- Moon orbits Earth
- While the Earth rotates around its axis, the Moon moves relative to the Earth, so
- 24 hours 50 minutes for observer to see subsequent Moons directly overhead (lunar day)
- High lunar tides are 12 hours and 25 minutes apart → period of principal lunar semidiurnal tide is 12 h 25 m
- 24 hours for observer to see subsequent Suns directly overhead (solar day)
- High solar tides are 12 hours apart → period of principal solar semidiurnal tide is 12 h

Lunar day versus solar day (Trujillo and Thurman)

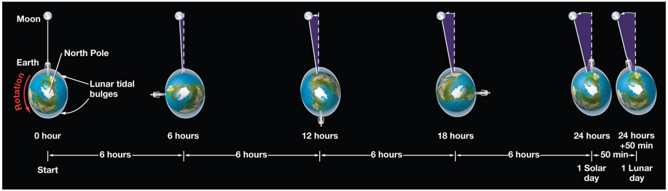

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Fig. 9.7, p. 281

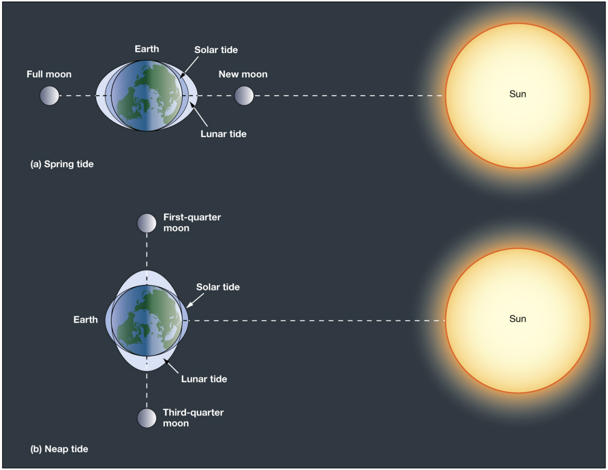
Equilibrium tide IV: Spring tide (Trujillo and Thurman)

Spring tides occur when the Moon, Earth and Sun are aligned (in syzygy, upper diagram on next page). Either new moon or full moon. Tidal range (difference between high and low tides) is large: exceptionally high high tides, low low tides.

Neap tides occur when Earth-Moon line is at right angles to Earth-Sun line (lower diagram on next page). Sun's bulges do not reinforce the Moon's bulges then. Tidal range is lower; high tides not so high, and low tides not so low. Occurs during first-quarter or third-quarter moon.

Spring tides occur twice a (lunar) month (not during Spring Season!!). Similarly, the neap tides occur twice a (lunar) month; twice each 29.5 days.

Spring tides continued (Trujillo and Thurman)



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Equilibrium tide V: Diurnal tides

- Diurnal tides are due to the declination of the Moon's orbit to the equator.

- Declination yields asymmetries in the two high tides that occur every day.

- Mathematically, this is like having a forcing at once per day.

Declination

- Angular distance of Moon or Sun above or below Earth's equator
- Sun to Earth: 23.5º N or S of equator
- Moon to Earth: 28.5º N or S of equator

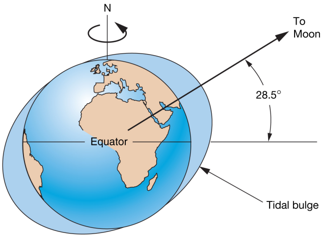

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Fig. 9.11, page 284

- Shifts lunar and solar bulges from equator
- Diurnal inequality: Two high tides a day are of unequal size

→ Tidal forcing exists at periods near 24 h

Harmonic decomposition

- Frequencies of celestial motions:
  - $\omega_0 = \frac{2\pi}{1 \; mean \; solar \; day}$
  - $\omega_1 = \frac{2\pi}{1 \; mean \; lunar \; day} = \frac{2\pi}{1.0351 \; mean \; solar \; days}$
  - $\omega_2 = \frac{2\pi}{1 \; sidereal \; month} = \frac{2\pi}{27.3217 \; mean \; solar \; days}$
  - $\omega_3 = \frac{2\pi}{1 \; tropical \; year} = \frac{2\pi}{365.2422 \; mean \; solar \; days}$
  - And others...
- Frequencies of some tidal constituents:
  - M_2: $2\omega_1$
  - S_2: $2\omega_0$
  - K_1: $\omega_{sidereal} = \omega_0 + \omega_3 = \omega_1 + \omega_2$
  - M_f: $2\omega_2$
  - And many others...
- Spring-neap cycle: frequency beating, especially M_2 and S_2

Tidal species

- The latitudinal and longitudinal dependence of the equilibrium tide η_{EQ} depends on the "tidal species" involved.
    - Semidiurnal tides (M_2,S_2,N_2,K_2):
        - $\eta_{EQ} = A(1 + k_2 - h_2)cos^2(\phi)cos(\omega t + 2\lambda)$
    - Diurnal tides (K_1,O_1,P_1,Q_1):
        - $\eta_{EQ} = A(1 + k_2 - h_2)sin(2\phi)cos(\omega t + \lambda)$
    - Long-period tides (M_f, M_m):
        - $\eta_{EQ} = A(1 + k_2 - h_2)[\frac{1}{2} - \frac{3}{2}sin^2(\phi)]cos(\omega t)$
- λ is longitude with respect to Greenwich
- ϕ is latitude
- t is time
- A and ω are constituent-dependent amplitudes and frequencies, respectively.
- The factor $1+k_2-h_2$ will be discussed later.

Ten commonly considered constituents

Const.	ω (10^{-4} s^{-1})	A (cm)	Period (solar days)
M_m	0.026392	2.2191	27.5546
M_f	0.053234	4.2041	13.6608
Q_1	0.6495854	1.9273	1.1195
O_1	0.6759774	10.0661	1.0758
P_1	0.7252295	4.6848	1.0027
K_1	0.7292117	14.1565	0.9973
N_2	1.378797	4.6397	0.5274
M_2	1.405189	24.2334	0.5175
S_2	1.454441	11.2743	0.5000
K_2	1.458423	3.0684	0.4986

Question: Can the actual ocean tide equal the equilibrium tide?

- Or, asking it in a different way: are there any factors you can think of that might prevent the actual ocean tides from attaining the equilibrium state?

Reasons for actual tides to be different from equilibrium tides

- Friction
- Earth's rotation
- Continents in the way
- Wave speed not fast enough (homework)

• The response is complex and varies from place to place.

• In fact, in some locations, the dominant tide is diurnal (next slide).

Tidal types (Trujillo and Thurman)

Monthly tidal curves

- During a month there are two spring tides and two neap tides at all locations.
- Boston has semi-diurnal tides
- San Francisco and Galveston have diurnal/mixed tides
- Pakhoi, China has diurnal tides

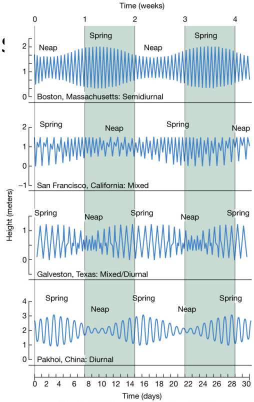

Fig. 9.16, p. 290

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Quantitative modeling of tidal response

• We model the tidal response to astronomical forcing with the shallow-water equations forced by the equilibrium tide.

• Such equations are traditionally known as the Laplace Tidal Equations.

Laplace's tidal equations

• Laplace: actual tide is dynamical response to equilibrium forcing.
• Modern shallow-water models often include nonlinearities and friction:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \bullet \nabla \vec{u} + f \hat{k} \times \vec{u} = -g \nabla(\eta - \eta_{EQ}) + \mathit{Friction}$$

$$\frac{\partial \eta}{\partial t} + \nabla \bullet [(H + \eta)\vec{u}] = 0$$

• In recent forward tide models, *Friction* includes eddy viscosity, quadratic bottom drag $\frac{c_d |\vec{u}| \vec{u}}{H}$, and a term having to do with energy conversion into internal waves over rough topography.
• Note that the equilibrium tide is the reference which the sea surface elevations η are measured against. The equilibrium tide is a perturbation to the geoid (equipotential).

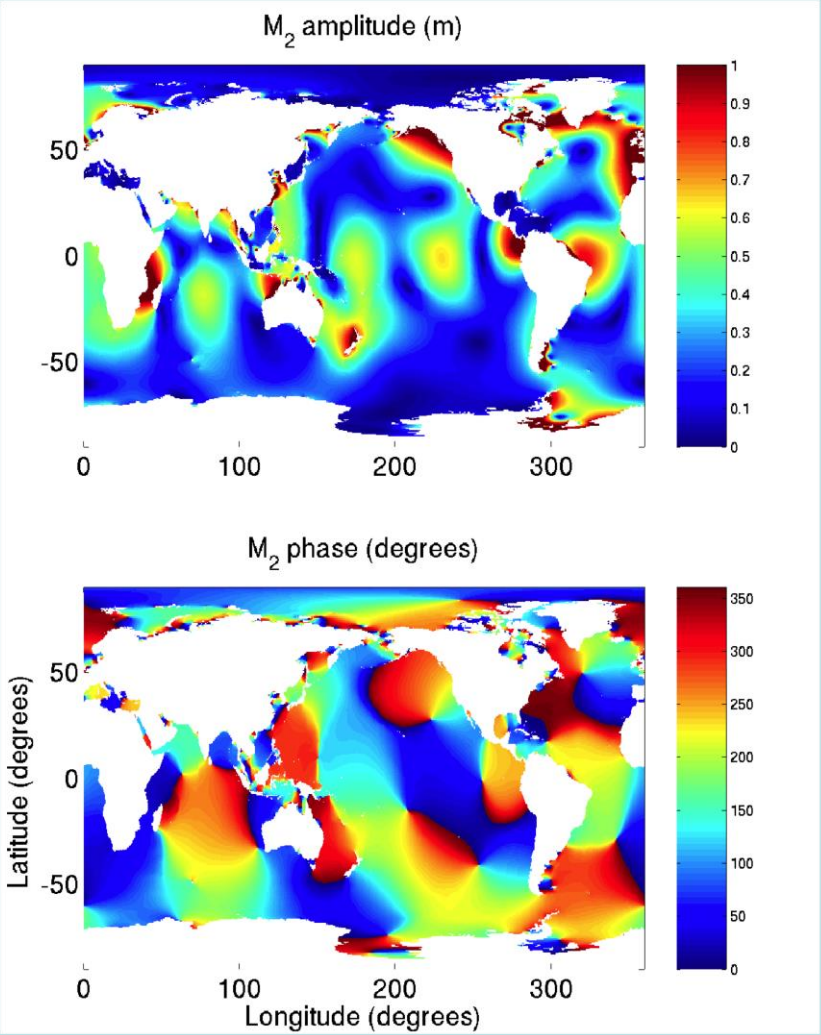
Tidal response

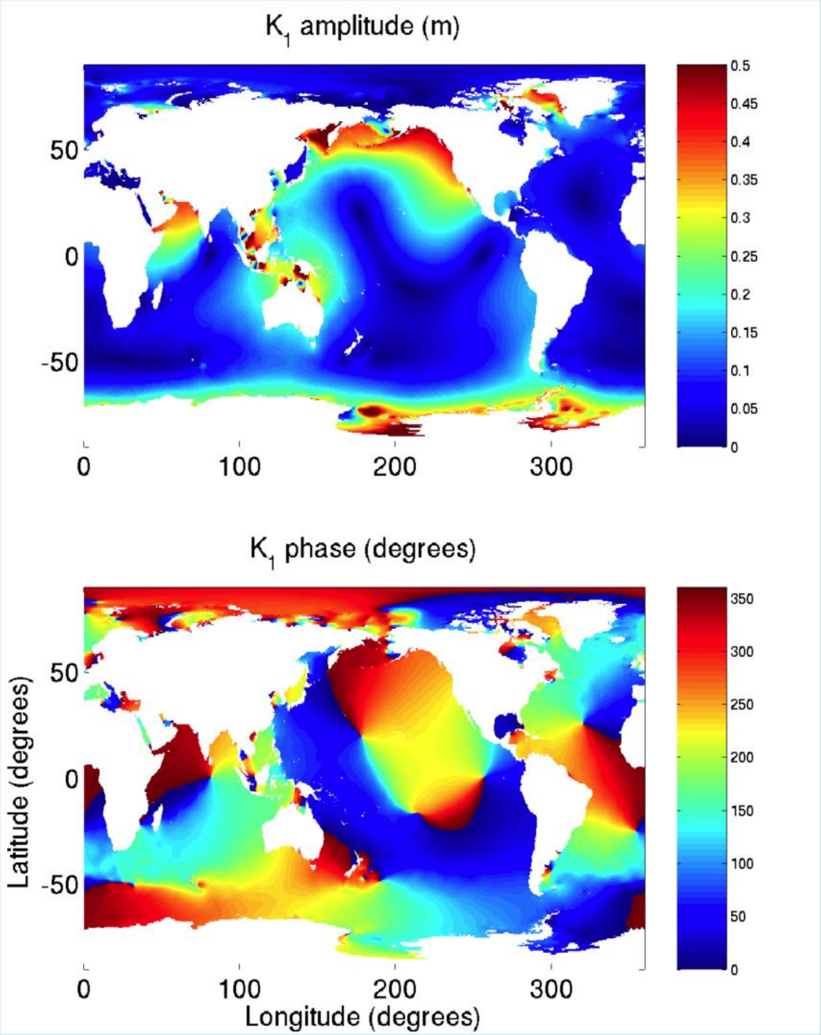
• For each constituent:
$Tidal\,elevation(\phi, \lambda) = Amplitude(\phi, \lambda) cos[\omega t - phase(\phi, \lambda)]$,
phase wrt Greenwich

• We can construct maps of the amplitude and phase of tidal elevations, for each constituent.

• Note spring tidal range is the *peak-to-peak* value, which is twice the sum of the amplitudes of several constituents.

The tidal patterns (mixed vs. semidiurnal vs. diurnal) can be understood by comparing the upper parts of these two plots:





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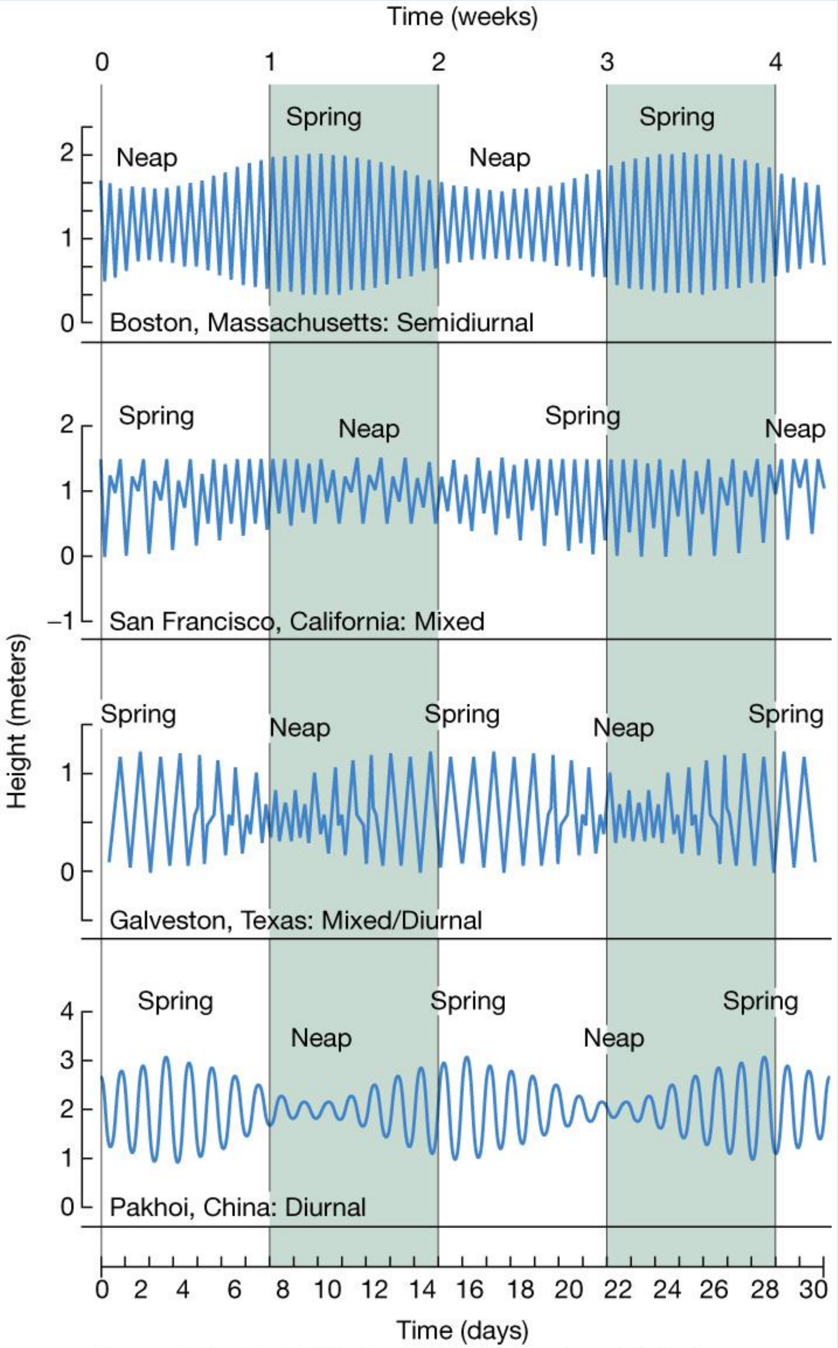

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Coastal tides

- The largest tidal elevations (height changes), and the largest tidal currents, are found in coastal areas.

- Typical tidal current values:
  - Up to $\sim$ 1 m s^{-1} in coastal areas
  - Typically about 2 cm s^{-1} in the open ocean

- Typical values of constituent elevation amplitudes:
  - Up to $\sim$ 4 m in coastal areas
  - Typically about 0.2-1 m in the open ocean

Large coastal tides (Trujillo and Thurman)

• Spring tidal range can be up to 17 m in regions of large tides such as the Bay of Fundy and the Hudson Strait.

• Note spring tidal range is the *peak-to-peak* value, which is twice the sum of the amplitudes of several constituents.

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